Math Workbook

Grade 11 CST

Term 1

Review Unit – Solving Algebraic Equations

1.1 SOLVING ALGEBRAIC EQUATIONS

Variables are letters or symbols that represent numbers we do not know or that can change. We can use algebra to find the exact value of the variable.

Ex: Ms. James brought some oranges to school. During the day, she gave away 2 oranges. At the end of the day, Ms. James had 3 oranges. How many oranges did Ms. James bring to school?

There are two keys to solving equations:

1) To get rid of a number, you do its opposite (for example, if it was addition, use subtraction).
2) Whatever you do to one side, you do the same to the other side.

Let's look at 5 different levels.

Level 1: Variable on one side with only addition or subtraction

Get rid of the addition or subtraction by doing the opposite on both sides.

Ex: Solve for x.

a) $x + 2 = 3$ b) $3 + x = 7$ c) $x - 4 = 8$ d) $-7 + x = 2$

Try these questions! Solve for x:

1a) $x - 6 = 10$ b) $8 + x = 11$ c) $2 = -5 + x$ d) $20 = x + 3$

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Level 2: Variable on one side with addition or subtraction and multiplication.

Get rid of the addition or subtraction first, then get rid of the multiplication by dividing.

Ex: Solve for x.

a) $3x + 9 = 12$ b) $4 + 2x = 10$ c) $20x - 25 = 75$ d) $-7 + 3x = 20$

Try these questions! Solve for x:

2a) $4x - 6 = 6$ b) $8 + 7x = 43$ c) $15 = -5 + 5x$ d) $33 = 10x + 3$

Level 3: Variables and addition or subtraction on both sides

Get rid of the variable on one side, then get rid of the addition or subtraction on the other side.

Ex: Solve for x.

a) $3x + 3 = 13 - 2x$ b) $4 - 4x = 18 - 6x$ c) $28x - 15 = 8x + 65$

Try these questions! Solve for x:

3a) $2x - 6 = 8 - 5x$ b) $8 + 7x = 3x + 28$ c) $15 - 2x = -5 + 2x$

Review Unit – Solving Algebraic Equations

Level 4: Brackets on one or both sides

Get rid of the brackets by expanding and then solve like in Level 3.

Ex: Solve for x.

a) $6(x-2)=15+3x$ b) $3(4+3x)=4(2+x)$ c) $2(7-8x)=5(2x-4)$

Try these questions! Solve for x:

4a) $2(x+3)=10-2x$ b) $7(2x+3)=3(4x-2)$ c) $3(2-5x)=5(4-6x)$

Level 5: Division (fractions) on one or both sides

Cross multiply then solve like Level 4.

Ex: Solve for x.

a) $\frac{3x+2}{4}=\frac{2-6x}{5}$ b) $\frac{4-x}{3}=\frac{5-2x}{2}$ c) $\frac{8x+3}{2}=2+3x$

Try these questions! Solve for x:

5 a) $\frac{2+4x}{3}=\frac{3x-1}{2}$ b) $\frac{2x+7}{2}=\frac{4-6x}{4}$ c) $8x+3=\frac{12x-5}{5}$

Review Unit – Solving Algebraic Equations

Practice Questions: Solve for x.

1) $x + 3 = 5$

2) $2x + 4 = x + 12$

3) $3(x + 4) = 2(5 - 2x)$

4) $\frac{2x-3}{4} = \frac{2+x}{5}$

5) $3x - 6 = 18$

6) $x + 1 = 4(2 + 4x)$

7) $\frac{18x+3}{6} = 5x - 2$

8) $4x - 3 = 13$

Review Unit – Solving Algebraic Equations
Answer Key

Questions in the Notes

1a) $x = 16$	**1b)** $x = 3$	**1c)** $x = 7$	**1d)** $x = 17$
2a) $x = 3$	**2b)** $x = 5$	**2c)** $x = 4$	**2d)** $x = 3$
3a) $x = 2$	**3b)** $x = 4$	**3c)** $x = 5$	
4a) $x = 1$	**4b)** $x = -13.5$	**4c)** $x = 0.93$	
5a) $x = 7$	**5b)** $x = -1$	**5c)** $x = -0.71$	

Practice Questions

1) $x = 2$	**2)** $x = 8$
3) $x = -0.29$	**4)** $x = 1.64$
5) $x = 8$	**6)** $x = -0.47$
7) $x = 1.25$	**8)** $x = 4$

Review Unit – Linear Equations

1.2 LINEAR EQUATIONS

Linear equations are generally written in the form $y = ax + b$, where

a is the slope (or rate of change)

b is the y-intercept (or initial value)

Identify the slope and y-intercept in the following equations

Ex: Find the slope and y-intercept of the following lines.

a) $y = 3x + 4$	b) $y = \frac{2}{3}x - 7$	c) $y = x + 10$	d) $y = -\frac{4}{3}x - 12$
slope:	slope:	slope:	slope:
y-intercept:	y-intercept:	y-intercept:	y-intercept:

Sometimes we need to re-arrange the equation to get it in the form $y = ax + b$ before we can identify the slope and the y-intercept.

Ex: Find the slope and y-intercept of the following lines.

a) $2y = 8x + 4$	b) $4y - 5x = 12$	c) $3x - 7y = 28$	d) $-2x = 3y + 8$
slope:	slope:	slope:	slope:
y-intercept:	y-intercept:	y-intercept:	y-intercept:

Try these questions! Find the slope and y-intercept of the following lines.

1a) $3x + 4y = 12$ b) $2x = 3y + 15$ c) $x + 2y = 10$ d) $-2y - 10x + 4 = 0$

Review Unit – Linear Equations

Graphing a line

When we are graphing a line:

$$a = slope = \frac{rise}{run}$$

b = y-intercept (where the line crosses the y-axis).

Remember:

If slope is positive, "rise" up.

If slope is negative, "rise" down.

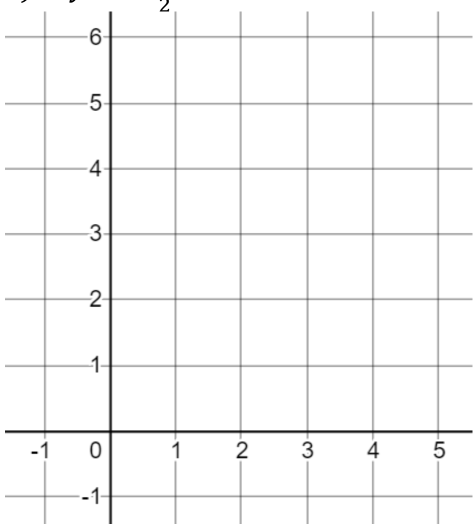
Always run right.

To graph a line:

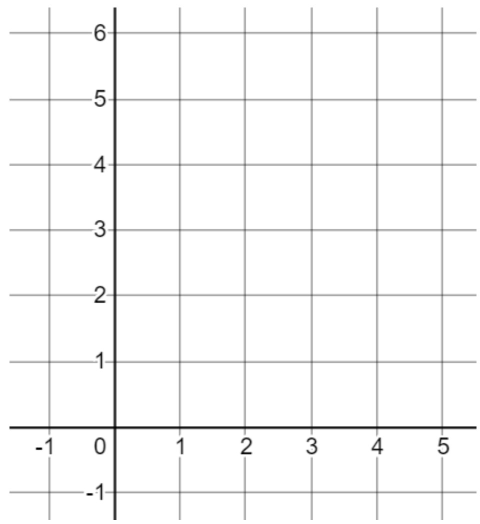
1) Put a dot on the y-axis at b.
2) Starting from b, use slope to find a second point. Put a dot there.
3) Connect the dots using a ruler.
4) Draw arrows at each end of the line.

Ex: Graph the following lines.

a) $y = -\frac{3}{2}x + 4$



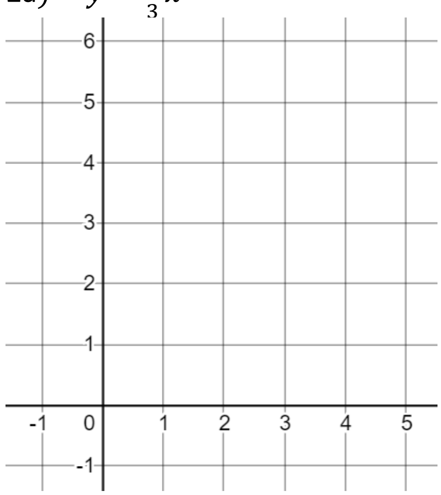
b) $y = 3x + 2$



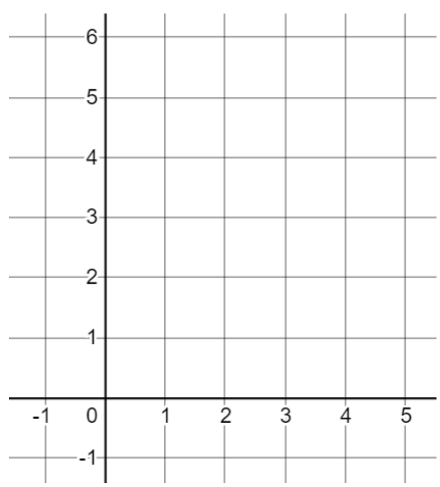
Remember:
If there is no b,
then $b = 0$

Try these questions! Graph the following lines.

2a) $y = \frac{4}{3}x$



b) $y = -2x + 5$

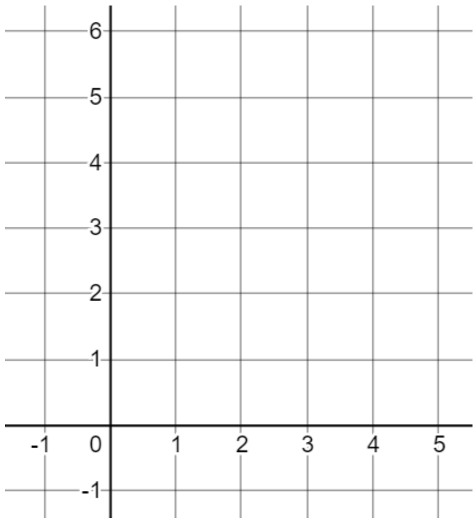


Review Unit – Linear Equations

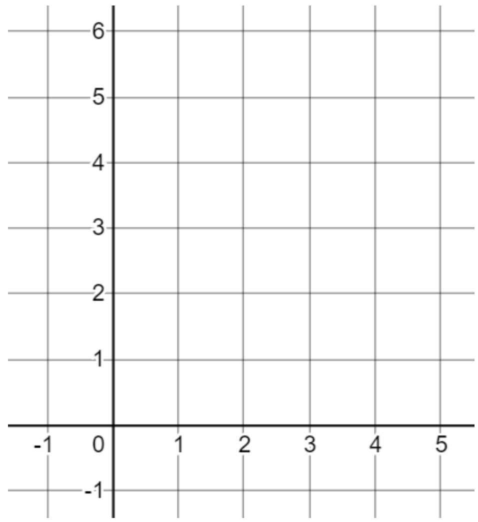
Sometimes we will have to re-arrange the equations before we can graph them.

Ex: Graph the following lines.

a) $x + y = 5$

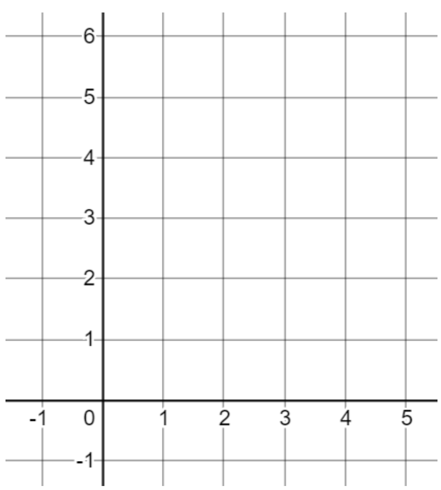


b) $2x = 3y - 3$

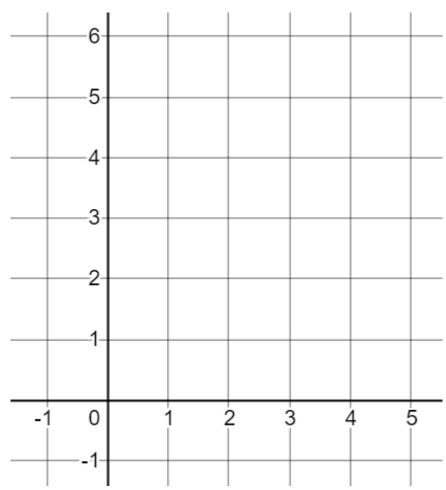


Try these questions! Graph the following lines.

3a) $5x + 2y = 10$



b) $x + y = 3$



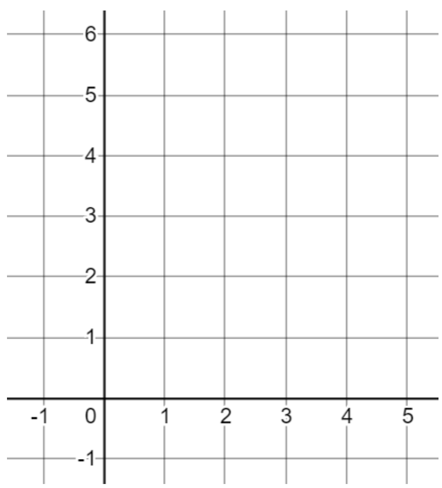
Sometimes we will have an equation of a line that only has one variable.

If the line only has an x (and not a y), put a dot on the x-axis at the number given and draw a vertical line.

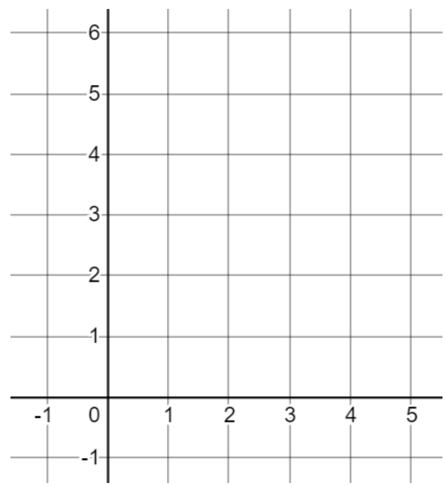
If the line only has a y (and not an x), put a dot on the y-axis at the number given and draw a horizontal line.

Ex: Graph the following lines.

a) $x = 2$

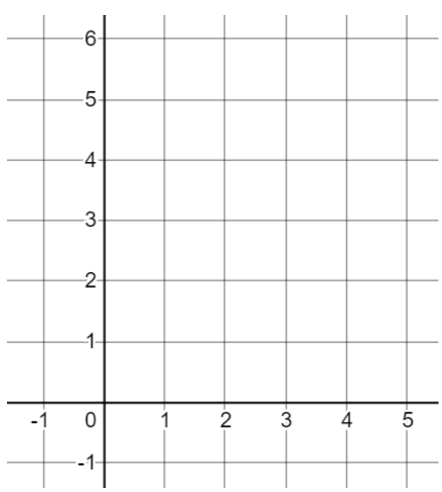


b) $y = 3$

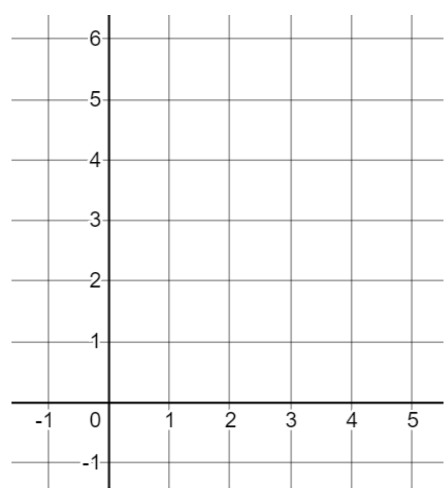


Try these questions! Graph the following lines.

4) $y = 5$



b) $x = 4$



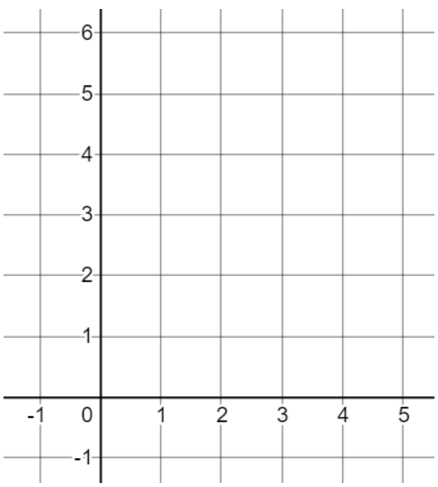
Review Unit – Linear Equations

Practice Questions

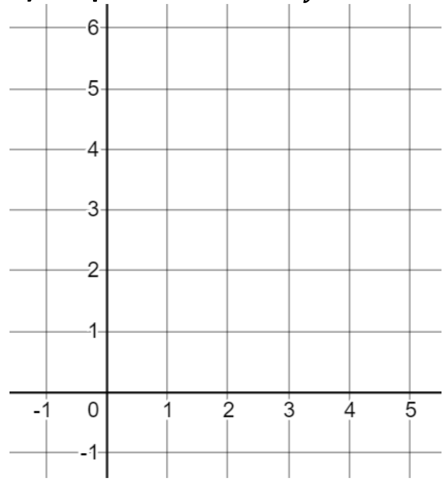
1) Find the slope and y-intercept of the line:
$y = \frac{2}{3}x - 10$

2) Find the slope and y-intercept of the line:
$3x - 7y = 35$

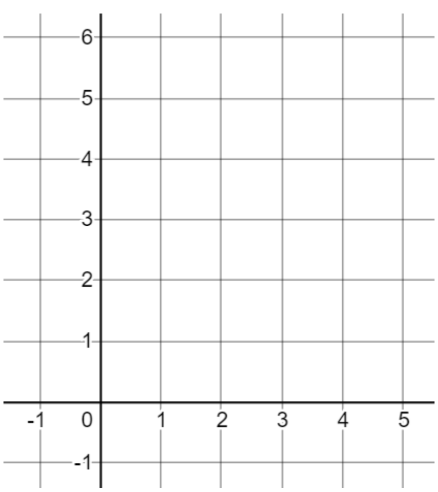
3) Graph the line: $y = 3x - 1$



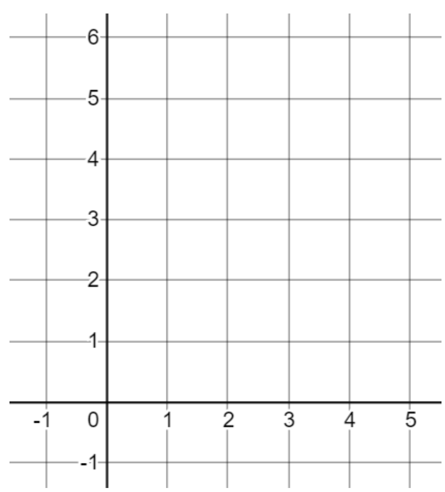
4) Graph the line: $3y + x = 15$



5) Graph the line: $x = 1$



6) Graph the line: $y = 4$



Review Unit – Linear Equations

Answer Key

Questions in the Notes

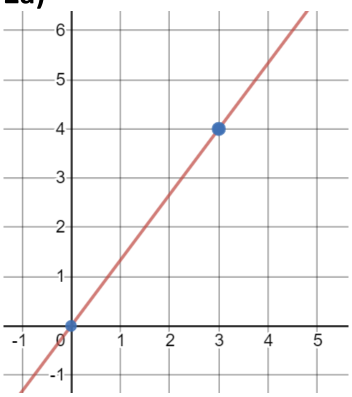
1a) slope: $-\frac{3}{4}$
y-intercept: 3

1b) slope: $\frac{2}{3}$
y-intercept: -5

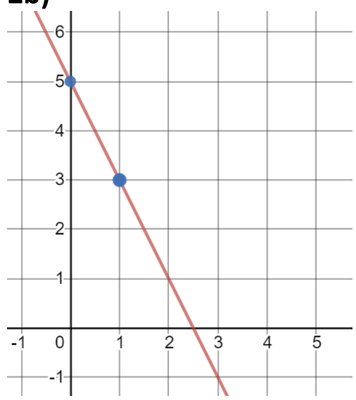
1c) slope: $-\frac{1}{2}$
y-intercept: 5

1d) slope: -5
y-intercept: 2

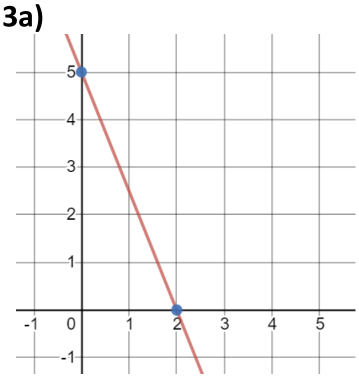
2a)



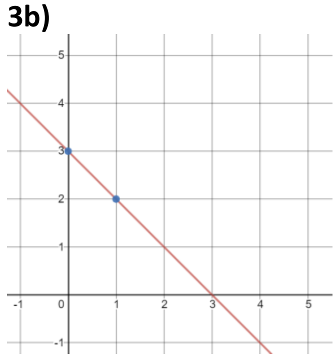
2b)



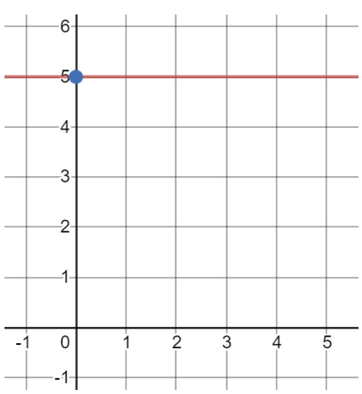
3a)



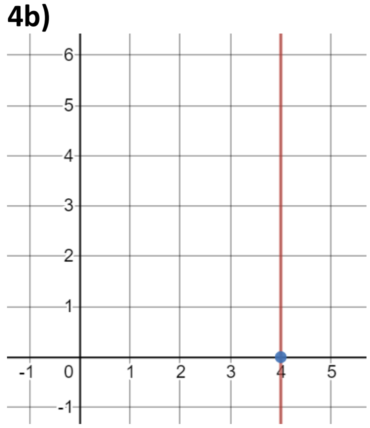
3b)



4a)



4b)



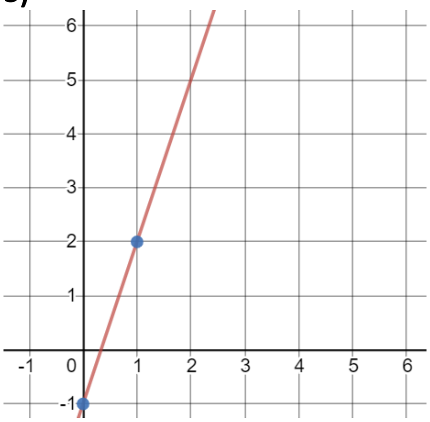
Review Unit – Linear Equations

Practice Questions

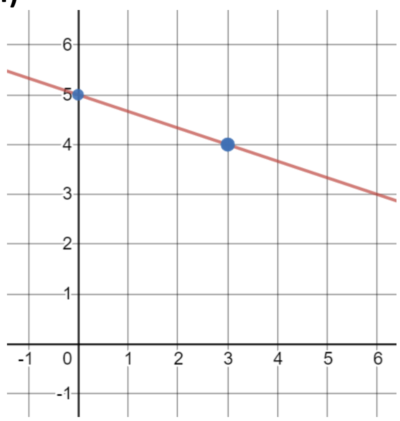
1) slope: $\frac{2}{3}$
y-intercept: -10

2) slope: $\frac{3}{7}$
y-intercept: -5

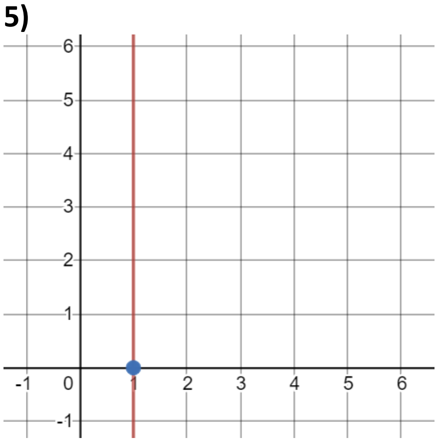
3)



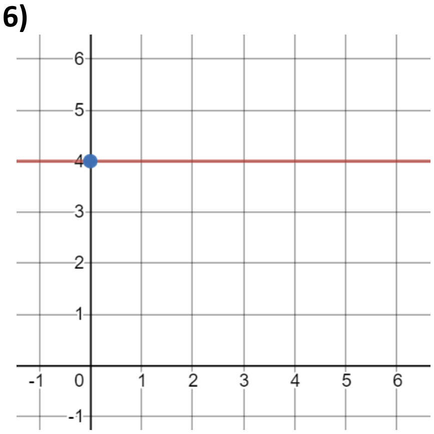
4)



5)



6)



Review Unit – Linear Systems

1.3 LINEAR SYSTEMS

A linear system is when we have two lines.

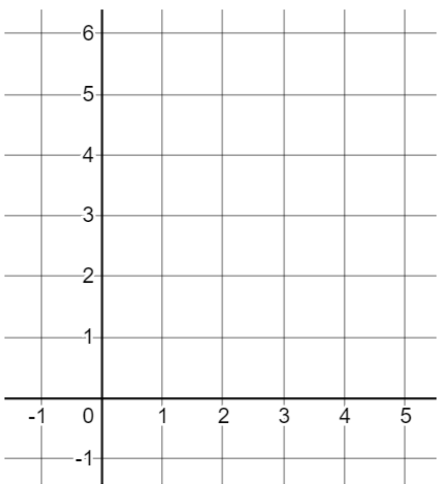
The **solution** is the point (x, y) where the two lines cross each other. We can find the solution to a system of equations graphically or algebraically (using elimination, comparison, or substitution).

Using a graph to solve:

The find a solution, graph both lines. The solution is the point where the lines cross, written as an ordered pair (x, y). Remember, you may need to re-arrange the equation before you can graph it.

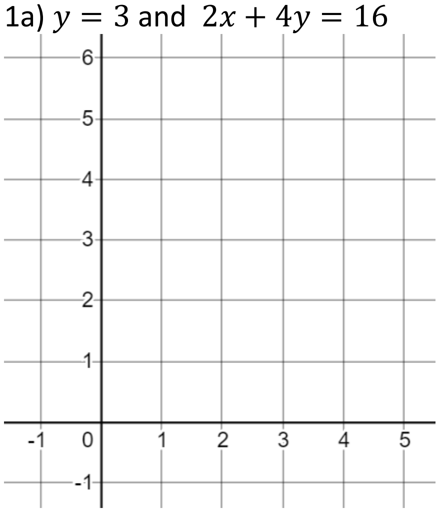
Ex: Find the solution to the linear system.

$y = 2x + 1$ and $x + y = 4$

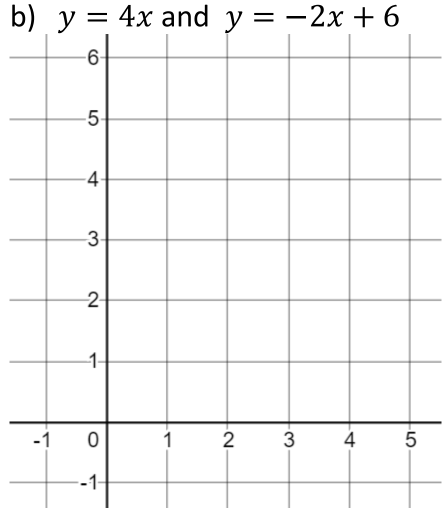


Try these questions! Find the solution to the linear systems.

1a) $y = 3$ and $2x + 4y = 16$



b) $y = 4x$ and $y = -2x + 6$



The graphing method is not an accurate way to solve linear systems. For example, it is difficult to tell the different between $(2.3, 4.6)$ and $(2.2, 4.7)$. Therefore, we will use algebra to solve linear systems.

There are 3 methods we can use: elimination, comparison, and substitution.

Using the elimination method to solve:

- Both lines must be in the form $Ax + By = C$.
- Multiply the entire first equation by the coefficient of x in the second equation.
- Multiply the entire second equation by the coefficient of x in the first equation, but change the sign.
- Add the two equations.
- Solve for the remaining variable.
- Use the solution in either equation to solve for the other variable.
- Write the solution (x, y).

Ex: Find the solution to the linear systems.

a) $2x + 5y = 16$ and $3x - 4y = 1$

b) $4x - 5y = 10$ and $y = -\frac{5}{3}x + 35$

Review Unit – Linear Systems

Try these questions! Find the solution to the linear systems.

2a) $2x + 5y = -4$ and $3x - 2y = 13$

b) $3x + 4y = -6$ and $y = -2x + 1$

Using the comparison method to solve:

- Both lines must be in the form $y = ax + b$.
- Take the $ax + b$ pieces from each equation and set them equal to each other $ax + b = ax + b$.
- Solve for x.
- Use either equation (and the value of x you just found) to solve for y.
- Write the solution (x, y).

Ex: Find the solution to the linear systems.

a) $y = 2x + 1$ and $y = -1.5x + 4.5$

b) $y = -2x - 6$ and $5x + y = -3$

Review Unit – Linear Systems

Try these questions! Find the solution to the linear systems.

3a) $y = 2x + 5$ and $y = -4x + 11$

b) $y = 0.5x + 2$ and $y - 2x = -1$

Using the comparison method to solve:

- This method works best if we already know the value of x or y.
- Use the equation that has both variables and replace the known variable.
- Solve for the missing variable.
- Write the solution as (x, y).

Ex: Find the solution to the linear systems.

a) $x = 2$ and $y = 3x + 8$

b) $y = 3$ and $3x + 4y = 20$

Try these questions! Find the solution to the linear systems.

4a) $y = 5$ and $y = 2x - 15$

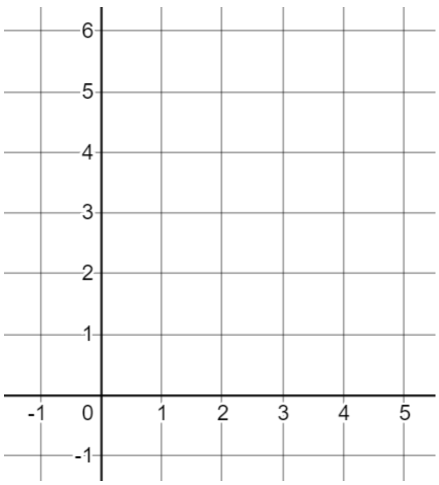
b) $x = 4$ and $3x + 2y = 20$

Review Unit – Linear Systems

Practice Questions

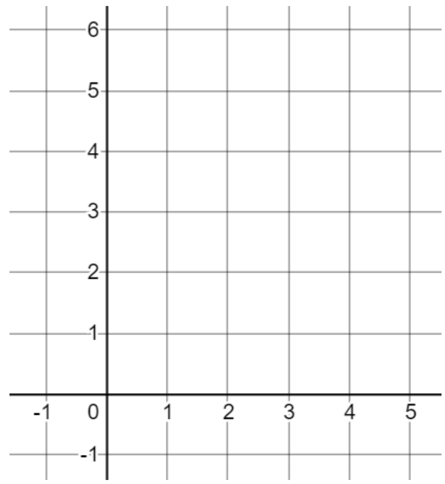
1) Solve the system using graphing:

$y = 4x - 10$ and $y = \frac{1}{3}x + 1$



2) Solve the system using graphing:

$y = -3x + 4$ and $y + 2 = 3x$



3) Solve the system using elimination:

$8x - 6y = -20$ and $-16x + 7y = 30$

4) Solve the system using elimination:

$-4y - 11x = 36$ and $20 = -10x - 10y$

5) Solve the system using comparison:
$y = x - 13$ and $y = -2x + 5$

6) Solve the system using comparison:
$y = -4x + 2$ and $x - y = 3$

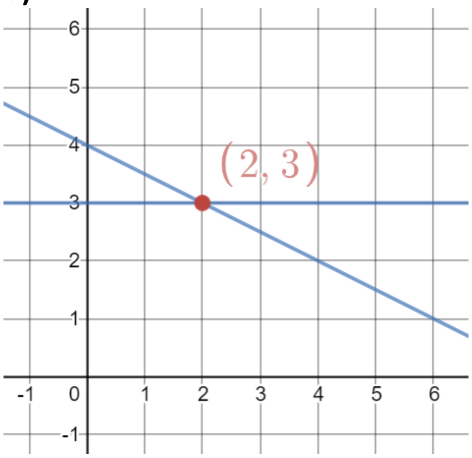
7) Solve the system using substitution:
$y = -5$ and $5x + 4y = -20$

8) Solve the system using substitution:
$x = 3$ and $4x - y = 20$

Review Unit – Linear Systems
Answer Key

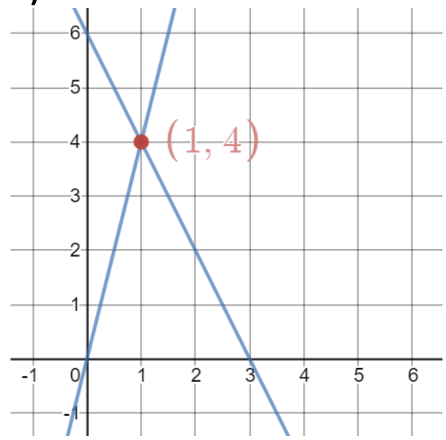
Questions in the Notes

1a)



The solution is $(2, 3)$

1b)



The solution is $(1, 4)$

2a) The solution is $(3, -2)$

2b) The solution is $(2, -3)$

3a) The solution is $(1, 7)$

3b) The solution is $(2, 3)$

4a) The solution is $(10, 5)$

4b) The solution is $(4, 4)$

Practice Questions

1) The solution is $(3, 2)$

2) The solution is $(1,1)$

3) The solution is $(-1, 2)$

4) The solution is $(-4, 2)$

5) The solution is $(6, -7)$

6) The solution is $(1, -2)$

7) The solution is $(0, -5)$

8) The solution is $(3, -8)$

2.1 OPTIMIZATION DEFINITION AND STEPS

Mathematics can help us figure out optimal solutions. Linear optimization will help us determine how to achieve maximum profit or minimum cost (for example).

Optimization is a long process, but we'll take it one step at a time.

Steps for Optimization:

1. Define variables ("Let x be… Let y be…")
2. Turn words into inequalities
3. Rearrange into $y = ax + b$ form
4. Graph and shade all inequalities
5. Identify polygon of constraints
6. Find (x, y) of each vertex (corner) of the polygon of constraints
7. Write optimizing function
8. Use optimizing function at each vertex
9. Choose maximum or minimum (depending on the question)
10. Write concluding sentence

2.2 OPTIMIZATION DEFINE VARIABLES AND TURN WORDS INTO INEQUALITIES

Step 1

The first step of optimization is defining our variable. To do this, we will read through the question and figure out the two things we're talking about.

Once we know the two things we care about, we will write two statements:

- "Let x be…"
- "Let y be…"

Note: I typically use x and y as the variables. This will make it easier when we start graphing.
I also usually let x be the first variable mentioned in the question and y be the second variable mentioned, but the final answer won't change if you swap them.

Ex: Define the variables for the following statement. There are red and blue marbles in a jar.

Try these questions! Define the variables in each of the statements below.

1. A pencil case contains pens and pencils.

2. Students are holding a car wash where they wash cars and trucks.

3. A garden grows red tulips and white tulips.

Optimization Unit – Define Variables and Turn Words into Inequalities

Step 2

Now that we've defined our variables, that will be the first thing we do in every optimization question. The next step is to turn statements into inequalities.

First, we'll look at turning statements into equalities.

Option 1: We are given a total amount. We will write the equation: $x + y = total$

Ex: Define the variables and turn the statement into an equation.
A bag contains red and blue marbles. There are a total of 15 marbles in the bag.

Try these questions! Define the variables and turn the statement into an equation.

4. A garden grows roses and tulips. There are a total of 300 plants in the garden.

5. Students are holding a bake sale. They sell cookies and cupcakes. A total of 250 treats are sold.

6. John collects stuffed dinosaurs and stuffed bears. He has a total of 25 stuffed animals in his collection.

Option 2: We are given a comparison statement. When we write the equation, x will be on one side and y on the other.

Hint: When reading comparison statements, define variables. See which variable comes first in the comparison statement. Write that variable on the left and the other variable on the right. Then figure out which thing you have more of. The multiplication (or addition) will go with the OTHER variable.

Ex: Define the variables and turn the statement into an equation.
A school is selling strawberry plants and tomato plants as a fundraiser. They sell twice as many strawberry plants as flower baskets.

Ex: Define the variables and turn the statement into an equation.
Students are holding a fundraiser washing cars and trucks. They wash 10 more trucks than cars.

Try these questions! Define the variables and turn the statement into an equation.

7. A company sells road bikes and mountain bikes. They sell three times as many road bikes as mountain bikes.

8. Ms. James drinks tea and coffee. In any given week, she drinks 3 more teas than coffees.

9. A farm has goats and chickens. There are 5 times as many chickens as goats

Optimization Unit – Define Variables and Turn Words into Inequalities

To change statements into inequalities, we will follow the same steps as changing statements to equalities, but instead of $=$ we will use $>, \geq, <, \text{or} \leq$.

Math Inequality Symbols and Words	
$<$	**Less Than** Is under Is fewer
$>$	**Greater Than** Is more than Is greater Exceeds
$\leq$	**Less Than or Equal To** Is at most Has a maximum of Is not greater than Does not exceed (go over) Is not more than
$\geq$	**Greater Than or Equal To** Is at least Is not less than Is not under Has a minimum value of

When we are converting words to inequalities, this chart can help us determine which inequality to use.

Hint: when turning statements into inequalities, make sure you write the variables in the same order as they appear in the statement.

Ex: Define the variables and change the statement into an inequality.
A family more dogs than cats.

Ex: Define the variables and change the statement into an inequality.
A food truck sells hotdogs and hamburgers. Every day they sell at least 100 items.

Optimization Unit – Define Variables and Turn Words into Inequalities

Ex: Define the variables and change the statement into an inequality.

Reese collects international and domestic stamps. Reese has no more than twice as many domestic stamps as international stamps.

Try these questions! Define the variables and turn the statement into an inequality.

10. Students are selling bracelets and necklaces as a fundraiser. They sell at least 5 more bracelets than necklaces.

11. Sidney is making homemade soaps to sell at a local market. Sidney makes citrus scented soap and lavender scented soap. Sidney expects to sell fewer than 100 total soaps.

12. Ms. Stinger is selling large and small jars of honey. She can make no more than twice as many small jars as large jars.

Optimization Unit – Define Variables and Turn Words into Inequalities
Practice Problems

A manufacturing plant produces cars and truck. Define the variables and translate each of the following statements into equalities or inequalities.

a) A maximum of 200 vehicles are produced day.

b) The plant must produce at least 100 vehicles each day.

c) Fewer than twice as many cars as trucks are produced.

d) The plant produces at least 40 cars each day.

e) The number of trucks produced must not exceed 150.

Optimization Unit – Define Variables and Turn Words into Inequalities
Answer Key

Questions in the Notes

1)
Let x be the number of pens

Let y be the number of pencils

2)
Let x be the number of cars

Let y be the number of trucks

3)
Let x be the number of red tulips

Let y be the number of white tulips

4)
Let x be the number of roses

Let y be the number of tulips

$$x + y = 300$$

5)
Let x be the number of cookies

Let y be the number of cupcakes

$$x + y = 250$$

6)
Let x be the number of stuffed dinosaurs
Let y be the number of stuffed bears

$$x + y = 25$$

7)
Let x be the number of road bikes

Let y be the number of mountain bikes

$$x = 3y$$

8)
Let x be the number of teas

Let y be the number of coffees

$$x = y + 3$$

9)
Let x be the number of goats

Let y be the number of chickens

$$y = 5x$$

10)
Let x be the number of bracelets

Let y be the number of necklaces

$$x \geq y + 5$$

11)
Let x be the number of citrus soaps

Let y be the number of lavender soaps

$$x + y < 100$$

12)
Let x be large jars of honey

Let y be small jars of honey

$$y \leq 2x$$

Practice Problems

Let x be the number of cars
Let y be the number of trucks

a) $x + y \leq 200$
b) $x + y \geq 100$
c) $x < 2y$
d) $x \geq 40$
e) $y \leq 150$

Optimization Unit – Rearranging Inequalities

2.3 OPTIMIZATION REARRANGING INEQUALITIES

After turning statements into inequalities we need to graph those inequalities. Before we can do that, we must rearrange them into $y = ax + b$ form so we can find the initial value and the slope.

However, we are going to keep the inequality symbol $(<, >, \leq, or \geq)$. We can rearrange inequalities just like we would rearrange equations with $=$, unless we multiply or divide by a negative number.

Ex: Define the variables, turn the statement into an equation, and rearrange into $y = ax + b$ form.
A bag contains red and blue marbles. There are at least 8 marbles in the bag.

Ex: Define the variables, turn the statement into an inequality, and rearrange into $y = ax + b$ form.
A bag contains red and blue marbles. There are no more than twice as many red marbles as blue marbles.

Optimization Unit – Rearranging Inequalities

Try these questions!
Define the variables, turn the statement an inequality and, and rearrange into $y = ax + b$ form.

13. Students are holding a bake sale. They sell cookies and cupcakes. No more than 75 treats are sold.

14. John collects stuffed dinosaurs and stuffed bears. He has at least three times as many dinosaurs as bears.

15. John collects stuffed dinosaurs and stuffed bears. He fewer than half as many bears as dinosaurs.

If an inequality only has one variable (either x or y, but not both), make sure the variable is on the left side of the inequality.

Ex: Define the variables, turn the statement into an inequality, and rearrange into $y = ax + b$ form.
In a school, students choose between art and music. At least 200 students sign up for music.

Optimization Unit – Rearranging Inequalities

Practice Problems

A café is selling tea and coffee. Define the variables, translate each of the following statements into inequalities, and rearrange into $y = ax + b$ form.

a) The café sells no more than 700 drinks in a day.

b) Every day the café sells at least 350 coffees.

c) The café sells fewer than twice as many coffees as teas.

d) A maximum of 200 teas are sold each day.

Optimization Unit – Rearranging Inequalities

Answer Key

Questions in the Notes

1)
Let x be the number of cookies

Let y be the number of cupcakes

$$y \leq -x + 75$$

2)
Let x be the number of dinosaurs

Let y be the number of bears

$$y \leq \frac{1}{3}x$$

3)
Let x be the number of dinosaurs

Let y be the number of bears

$$y < \frac{1}{2}x$$

Practice Problems

Let x be the number of teas
Let y be the number of coffees

a) $y \leq -x + 700$
b) $y \geq 350$
c) y$< 2x$
d) $x \geq 200$

Optimization Unit – Graphing Inequalities

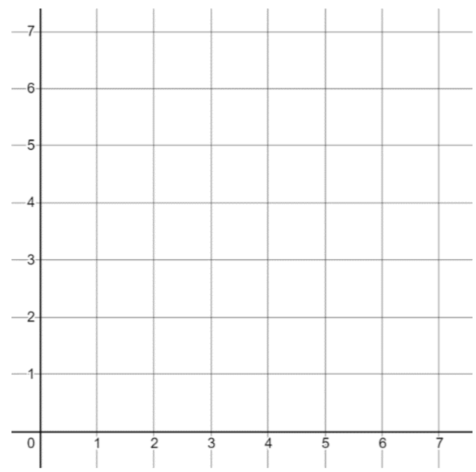
2.4 OPTIMIZATION GRAPHING INEQUALITIES

There are 3 steps to graphing an inequality (once it's been rearranged into $y = ax + b$ form).

1) Determine whether to use a solid line or a dotted line.
   a. Use a solid line if $\leq$ or $\geq$
   b. Use a dotted line if $<$ or $>$
2) Graph as usual (putting a dot on the y-axis at b and then using slope to find a second point).
3) Shade either above the line or below the line
   a. Shade above the line (draw arrows straight up) if $> or \geq$
   b. Shade below the line (draw arrows straight down) if $< or \leq$

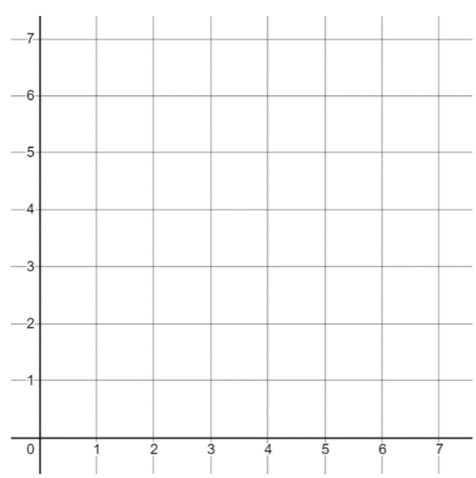
Ex: Define the variables, turn the statement into an equation, rearrange into $y = ax + b$ form and graph.

A bag contains red and blue marbles. There are at least 6 marbles in the bag.



Ex: Define the variables, turn the statement into an equation, rearrange into $y = ax + b$ form and graph.

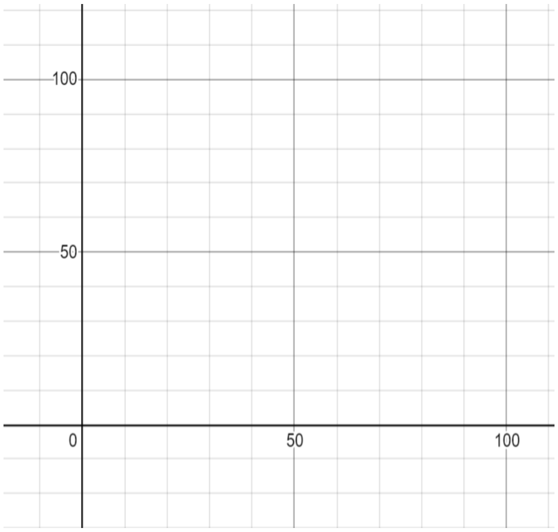
A bag contains red and blue marbles. There are more than twice as many red marbles as blue marbles.



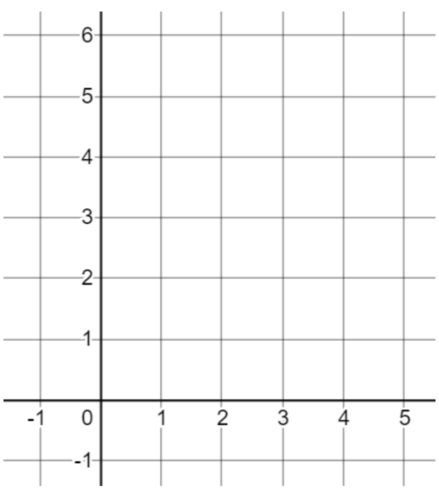
Optimization Unit – Graphing Inequalities

Try these questions!
Define the variables, turn the statement into an equation, rearrange into $y = ax + b$ form and graph.

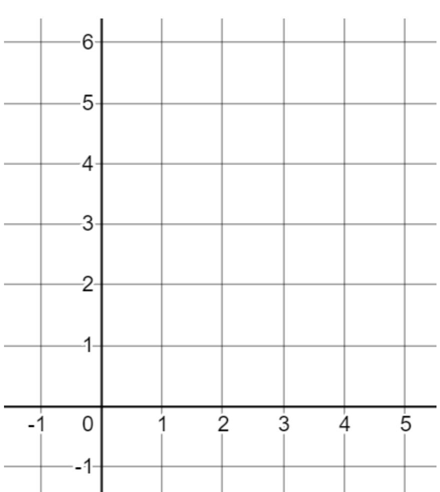
1. Students are holding a bake sale. They sell cookies and cupcakes. More than 75 treats are sold.



2. John collects stuffed dinosaurs and stuffed bears. He has at least three times as many dinosaurs as bears.



3. John collects stuffed dinosaurs and stuffed bears. He fewer than half as many bears as dinosaurs.



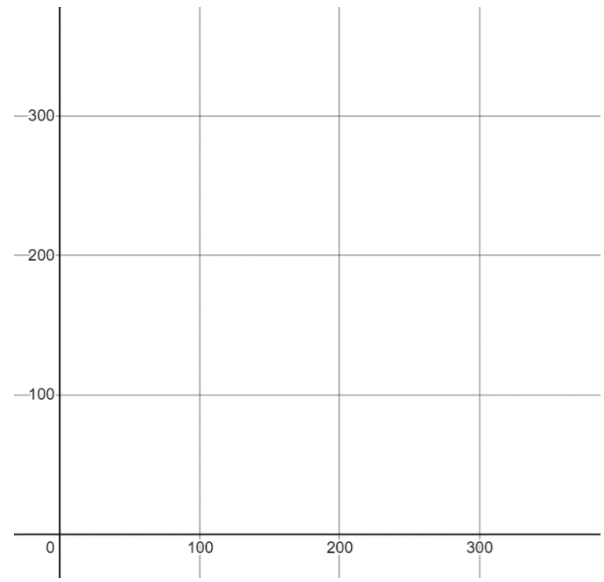
Optimization Unit – Graphing Inequalities

If you only have one variable, isolate the variable on the left side and graph and shade as usual, except:

- If $x >$ or $x \geq$ shade to the right of the line (or draw arrows to the right)
- If $x <$ or $x \leq$ shade to the left of the line (or draw arrows to the left)

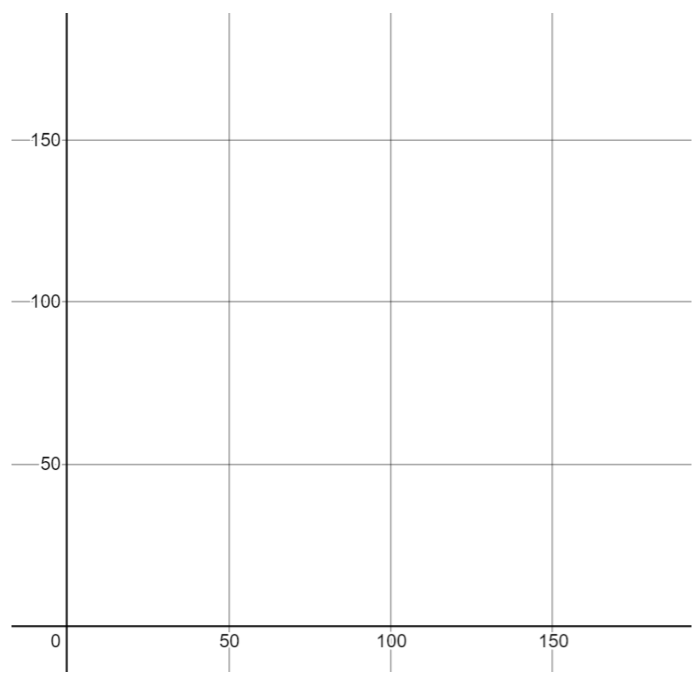
Ex: Define the variables, turn the statement into an inequality, and rearrange into $y = ax + b$ form.

In a school, students choose between art and music.
At least 200 students sign up for music.



Ex: Define the variables, turn the statement into an inequality, and rearrange into $y = ax + b$ form.

In a school, students choose between art and music. Fewer than 100 students sign up for art.

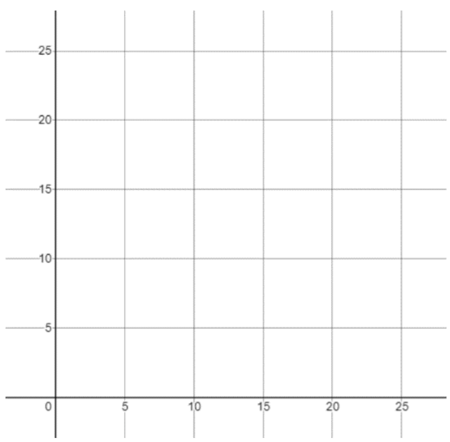


Practice Problems

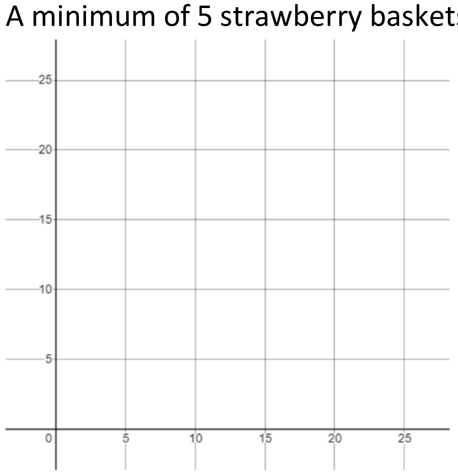
Define the variables, translate each of the following statements into inequalities, and rearrange into $y = ax + b$ form and graph for the scenario below.

Lisa, a Grade 11 student, is fundraising for Prom. She sells strawberry baskets and flower baskets.

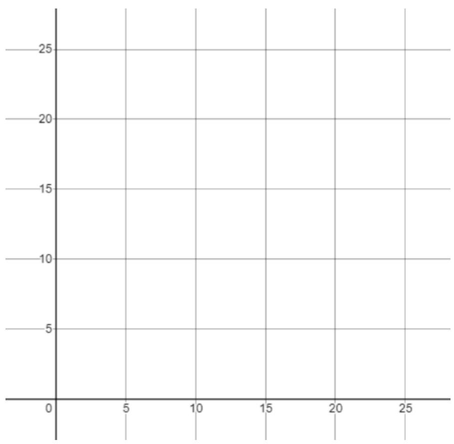
a) A maximum of 24 baskets can be sold.



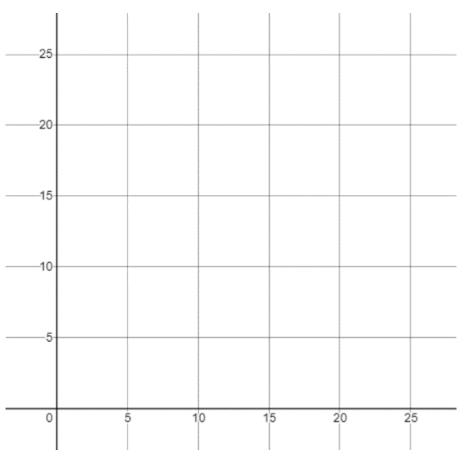
b) A minimum of 5 strawberry baskets must be sold.



c) The number of flower baskets must be at least triple the number of strawberry baskets sold.



d) A maximum of 20 flower baskets can be sold.

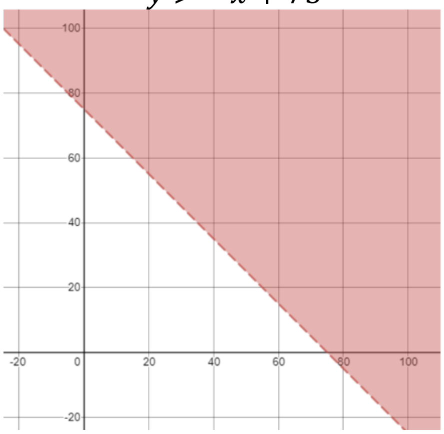


Optimization Unit – Graphing Inequalities
Answer Key

Questions in the Notes

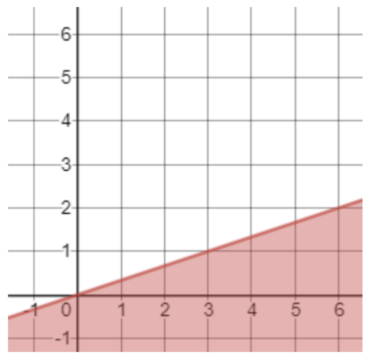
1)
Let x be the number of cookies
Let y be the number of cupcakes

$$x + y > 75$$
$$y > -x + 75$$



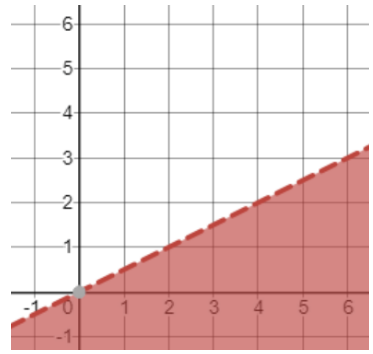
2)
Let x be the number of dinosaurs
Let y be the number of bears

$$3y \leq x$$
$$y \leq \frac{1}{3}x$$



3)
Let x be the number of dinosaurs
Let y be the number of bears

$$y < \frac{1}{2}x$$

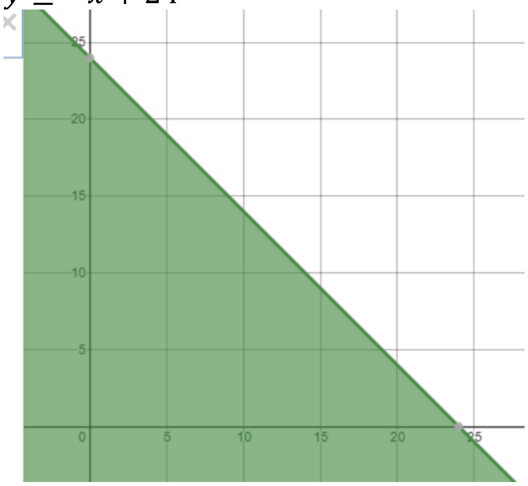


Optimization Unit – Graphing Inequalities

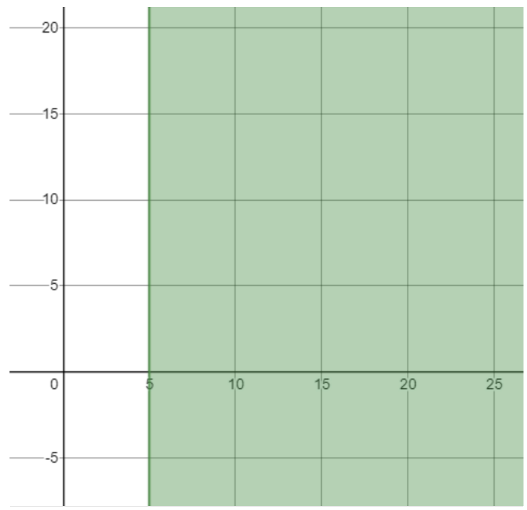
Practice Problems

Let x be the number of strawberry baskets
Let y be the number of flower baskets

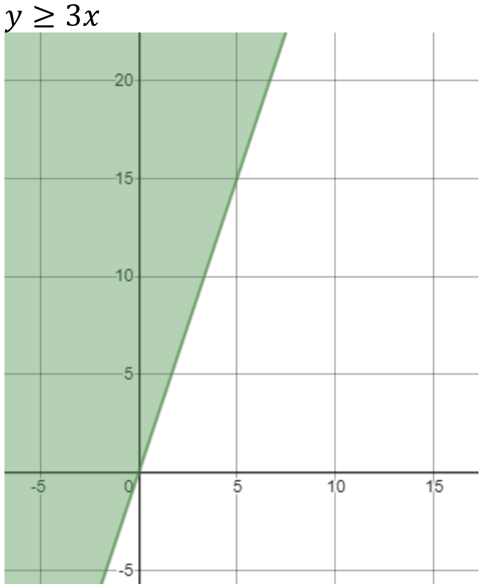
a) $x + y \leq 24$
$y \leq -x + 24$



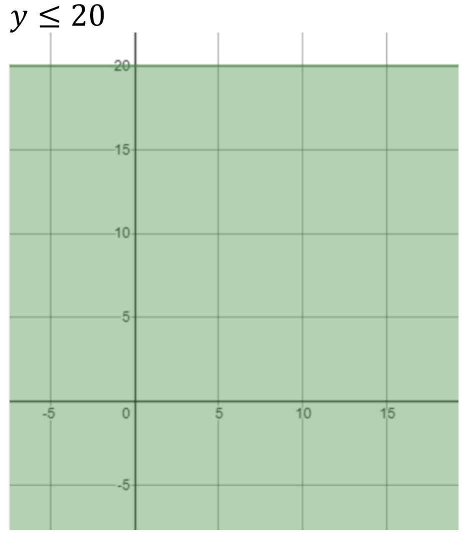
b) $x \geq 5$



c) $y \geq 3x$



d) $y \leq 20$



Optimization Unit – Polygon of Constraints

2.5 OPTIMIZATION POLYGON OF CONSTRAINTS

When we have more than one linear inequality, we can graph them all together. This is a system of linear inequalities.

The solution to a system of inequalities is where all the shading overlaps.

A **polygon of constraints** is when the overlapping shading is bound on all sides by a line.

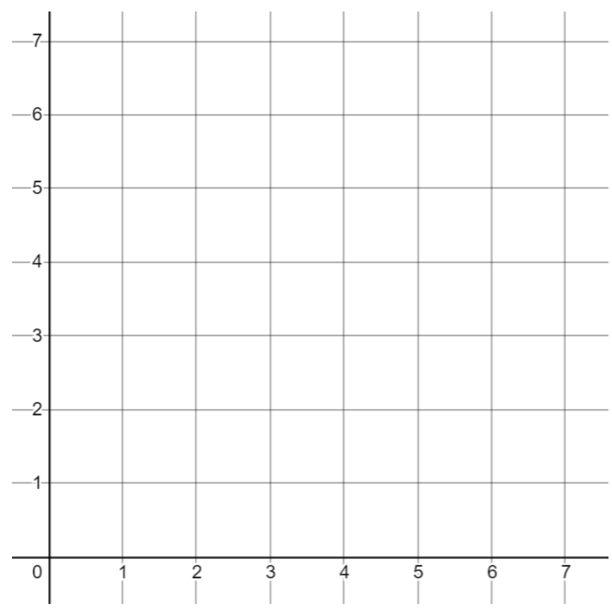
There are 3 steps to graphing a system of inequalities:

4) Graph and shade all lines (instead of shading, it can help to use arrows).
5) Determine the polygon of constraints by determining the area where all shading overlaps.
6) Shade the polygon of constraints.

Ex: Determine the polygon of constraints given the following scenario:

A bag contains red and blue marbles.

- There are no more than 4 blue marbles.
- There is at least 1 red marble.
- There is a maximum of 6 marbles.



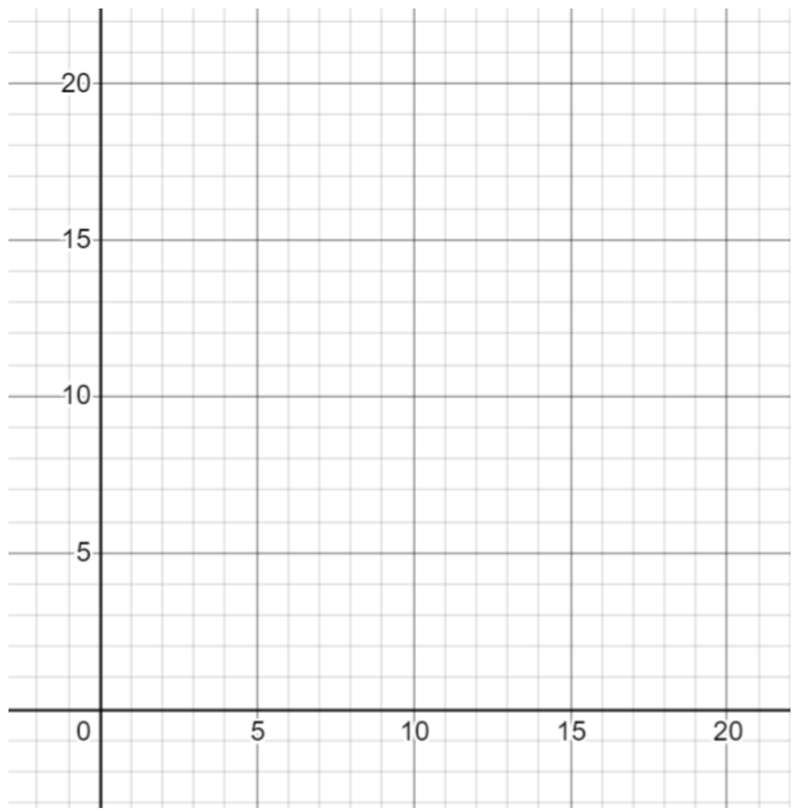
Optimization Unit – Polygon of Constraints

Try this question!

1) Determine the polygon of constraints given the following scenario.

John loves his stuffed animals. His collection has stuffed bears and stuffed dinosaurs.

- John has a minimum of 10 stuffed animals.
- John has a maximum of 20 stuffed animals.
- John has less than or equal to 6 bears.
- John has at least 5 more dinosaurs than bears.

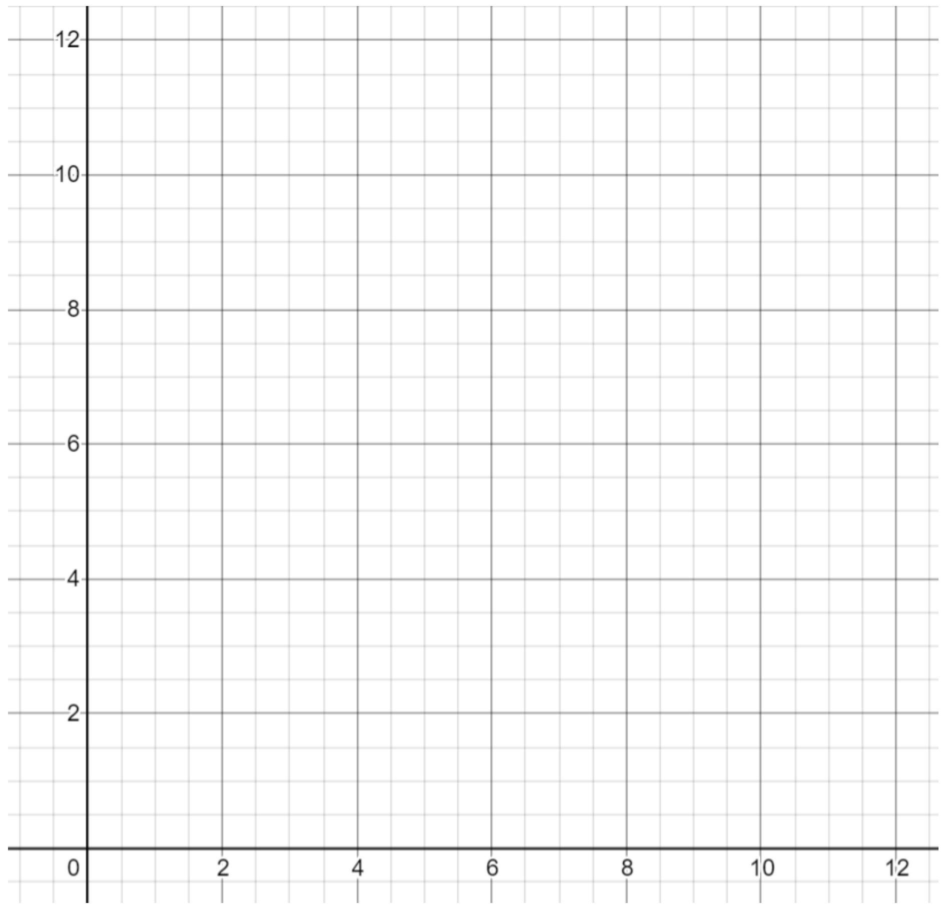


Optimization Unit – Polygon of Constraints

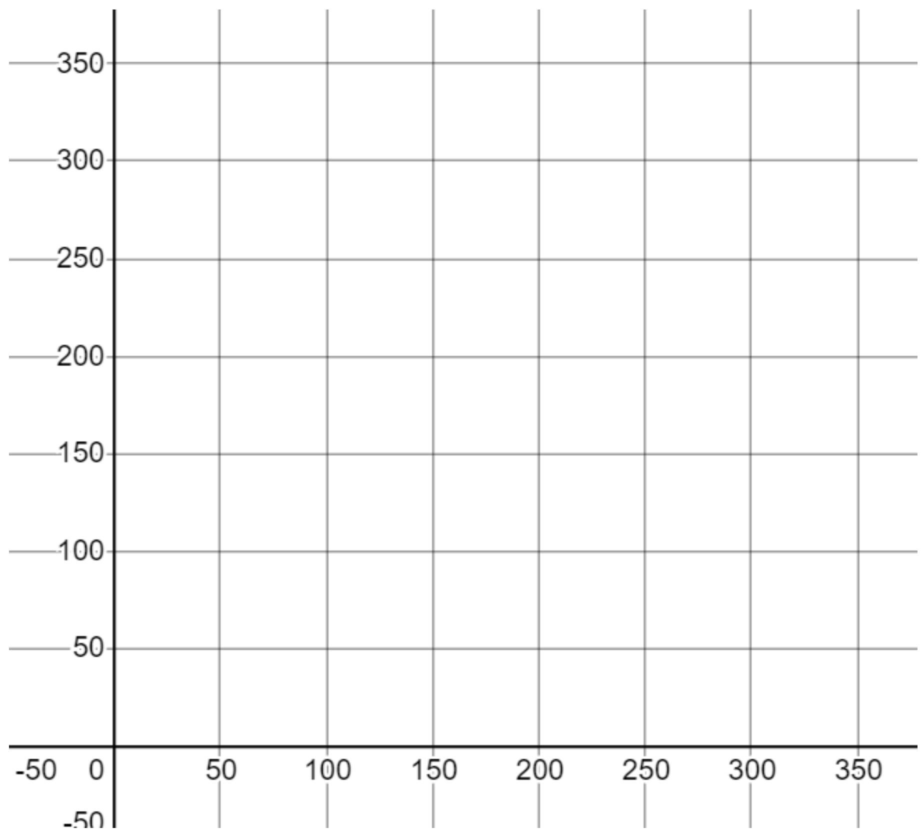
Practice Problems

Find the polygon of constraints for the following scenarios:

1) Sandra and Jane run a pet spa. They offer two services: dog grooming and cat grooming.
    - They will bathe a maximum of 12 cats per day.
    - They can bathe no more than 8 dogs per day.
    - They will bathe a maximum of 3 times as many cats as dogs.



2) To raise funds, members of an association organize a concert. They want to reserve some seats for donors and the other seats will be for general admission.
    - The theater contains a maximum of 300 seats.
    - There must be at least 3 times as many seats for general admission as there are seats reserved for donors.
    - There must be at least 50 seats for donors.
    - There is a maximum of 250 seats for general admission.



Optimization Unit – Polygon of Constraints

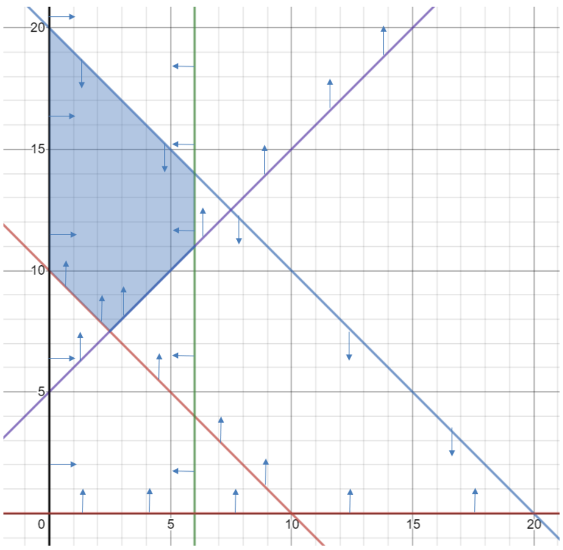
Answer Key

Questions in the Notes

1)

Let x be the number of stuffed bears

Let y be the number of stuffed dinosaurs

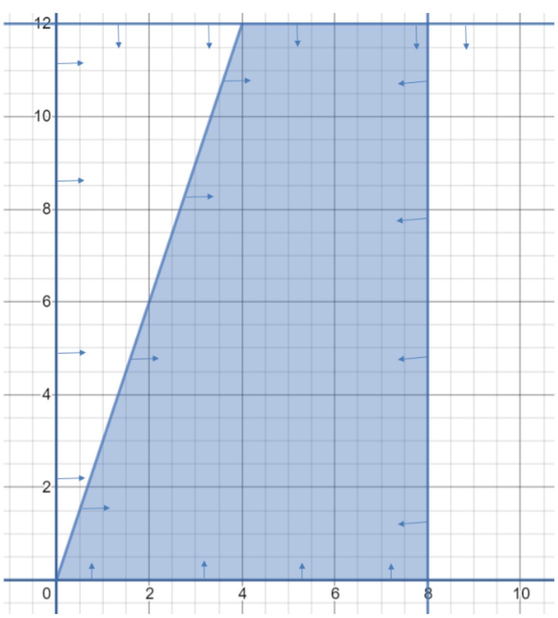


Practice Questions

1)

Let x be the number of dog baths

Let y be the number of cat baths

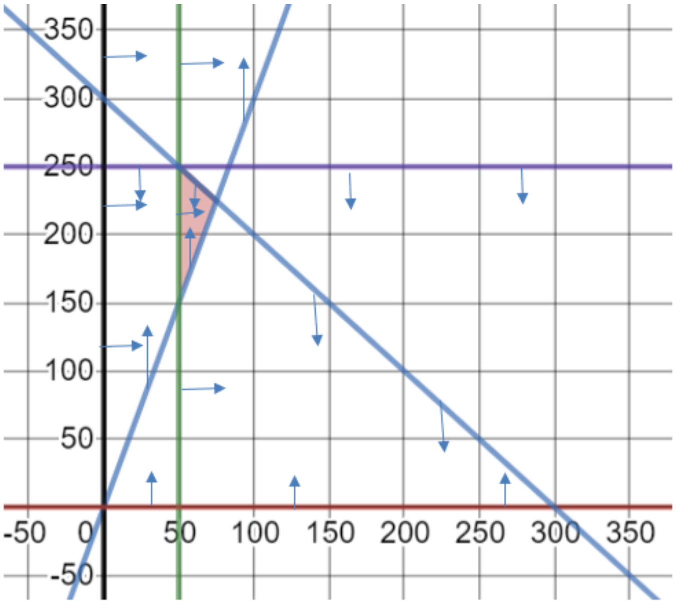


Optimization Unit – Polygon of Constraints

2)

Let x be the number of donor seats

Let y be the number of general admission seats



Optimization Unit – Finding Vertices

2.6 OPTIMIZATION FINDING VERTICES

Now that we have the polygon of constraints, we need to find the vertices. That is, we need to find the (x, y) of each vertex (corner) of the polygon of constraints.

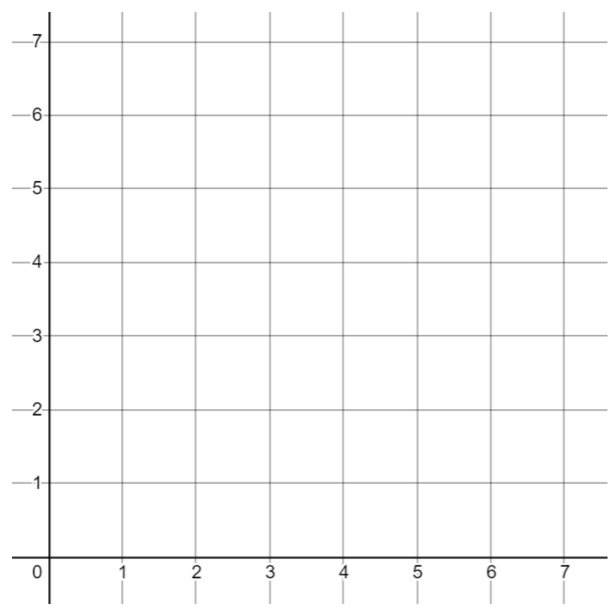
There are 3 steps to finding the vertices:

1) Label each vertex of the polygon of constraints (we usually use A, B, C, etc.).
2) Identify the two lines that go through each vertex and change inequalities to $=$
3) Solve the system

If the lines that intersect to form the vertex are:	Then use:
2 equations each including both variables	Comparison
1 equation with both variables and 1 equation with one variable	Substitution
2 equations each including one variable	No calculations necessary

Ex: Determine the vertices of the polygon of constraints given the following scenario:
A bag contains red and blue marbles.

- The bag contains a maximum of 6 marbles.
- The bag contains a minimum of 2 marbles.
- There are at least as many red marbles as blue marbles.
- The bag contains at most 5 red marbles.



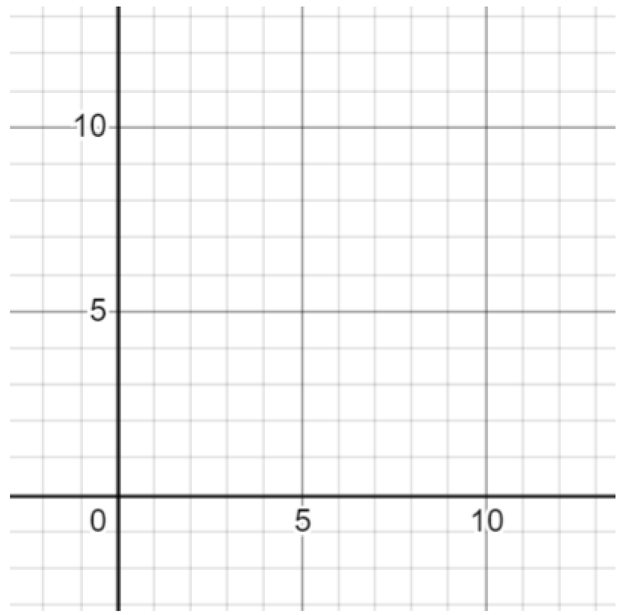
Optimization Unit – Finding Vertices

Try this question!

1) Determine the vertices of the polygon of constraints given the following scenario:

John loves his stuffed animals. His collection has stuffed bears and stuffed dinosaurs.

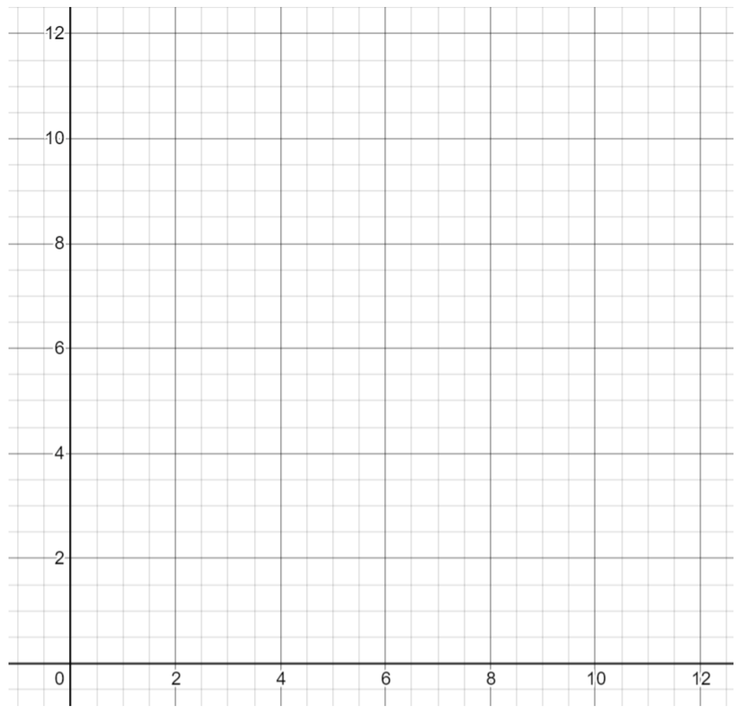
- John has a maximum of 10 stuffed animals.
- John has less than or equal to 4 bears.
- John has at least 1 more dinosaur than bear.



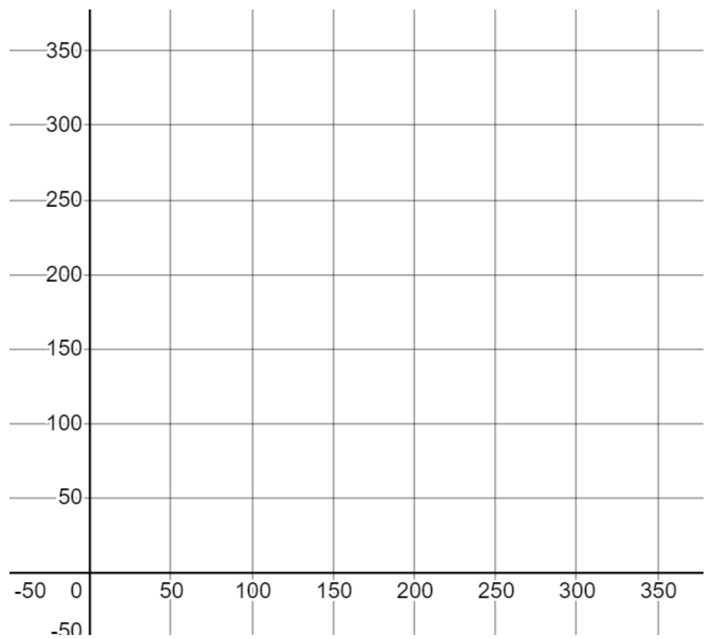
Optimization Unit – Finding Vertices

Practice Problems

3) Sandra and Jane run a pet spa. They offer two services: dog grooming and cat grooming.
    - They will bathe a maximum of 12 cats per day.
    - They can bathe no more than 8 dogs per day.
    - They will bathe a maximum of 3 times as many cats as dogs.



4) To raise funds, members of an association organize a concert. They want to reserve some seats for donors and the other seats will be for general admission.
   - The theater contains a maximum of 300 seats.
   - There must be at least 3 times as many seats for general admission as there are seats reserved for donors.
   - There must be at least 50 seats for donors.
   - There is a maximum of 250 seats for general admission.



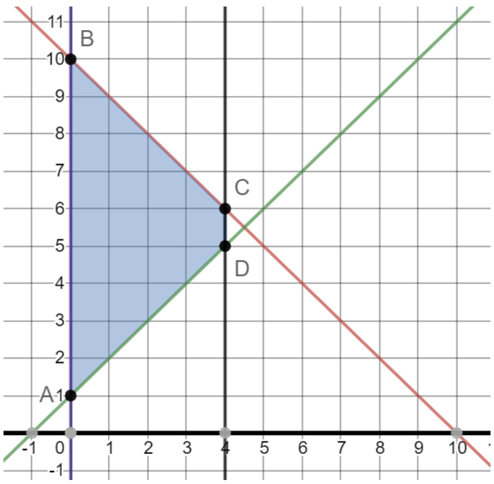
Optimization Unit – Finding Vertices
Answer Key

Questions in the Notes

1)

Let x be the number of stuffed bears

Let y be the number of stuffed dinosaurs



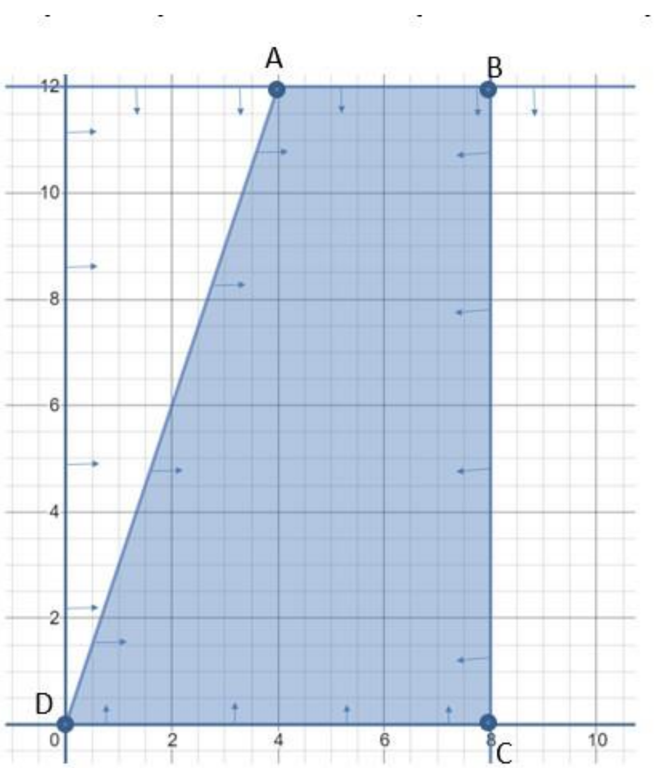
Vertex A	**(0, 1)**
Vertex B	**(0, 10)**
Vertex C	**(4, 6)**
Vertex D	**(4, 5)**

Practice Questions

1)

Let x be the number of dog baths

Let y be the number of cat baths



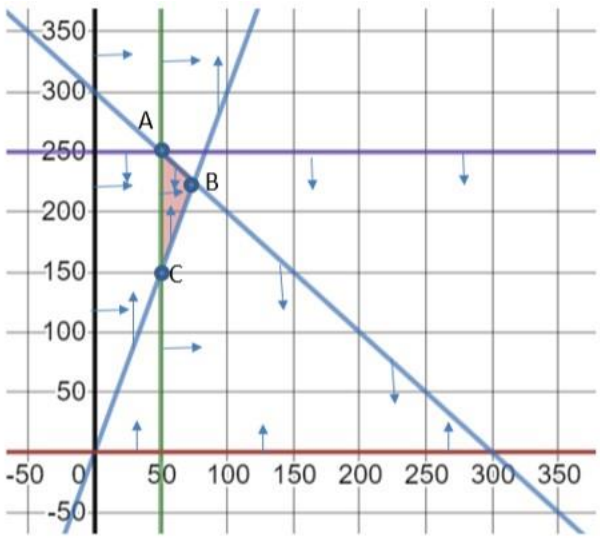
Vertex A	**(4, 12)**
Vertex B	**(8, 12)**
Vertex C	**(8, 0)**
Vertex D	**(0,0)**

Optimization Unit – Finding Vertices

2)

Let x be the number of donor seats

Let y be the number of general admission seats



Vertex A	**(50, 250)**
Vertex B	**(75, 225)**
Vertex C	**(50, 150)**

2.7 OPTIMIZATION TARGET OBJECTIVE, OPTIMIZING FUNCTION, AND ANSWERING THE QUESTION

The **target objective** is the search for the optimal solution. It is either the search for the highest value (maximum) or lowest value (minimum). The optimal value is obtained by using the **optimizing function**.

In each optimization question, you will be given a statement about money (cost, profit, etc.). This will become the optimizing function.

When you substitute each vertex into the optimizing function (one at a time), one vertex will give you the maximum and one vertex will give you the minimum.

- **Step 7** is to identify the target objective and write the optimizing function. Go back to the question and pick out the sentence that has to do with money. Turn this into an equation.
  **Be Careful! You won't have to use a statement about money until this step, so ignore it when you are creating your graph and finding your vertices.*

- **Step 8** is to use the optimizing function at each vertex. Take each vertex, one at a time, and replace the x and y in the optimizing function with the (x, y) coordinates of the vertex.

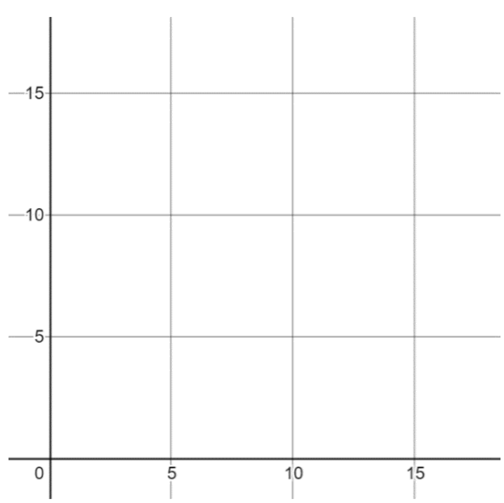
- **Step 9** is to choose the maximum or minimum depending on the vertex. One of the vertices will give you the maximum. One of the vertices will give you the minimum.

- **Step 10** is to write a concluding statement answering the question. The question may ask of the maximum, minimum, or the number of items that give you a maximum or minimum.

Ex: Determine the vertices of the polygon of constraints given the following scenario:
A bag contains red and blue marbles.

- The bag contains a maximum of 12 marbles.
- The bag contains a minimum of 6 marbles.
- There are no more than twice as many red marbles as blue marbles.
- The bag contains at least 3 red marbles.

If each red marble is work $1.50 and each blue marble is worth $3.00, what is the maximum value of the collection of marbles in the bag? How many of each marble do you need to have the maximum value?



Optimization Unit – Target Objective, Optimizing Function, and Answering the Question

Optimization Unit – Target Objective, Optimizing Function, and Answering the Question

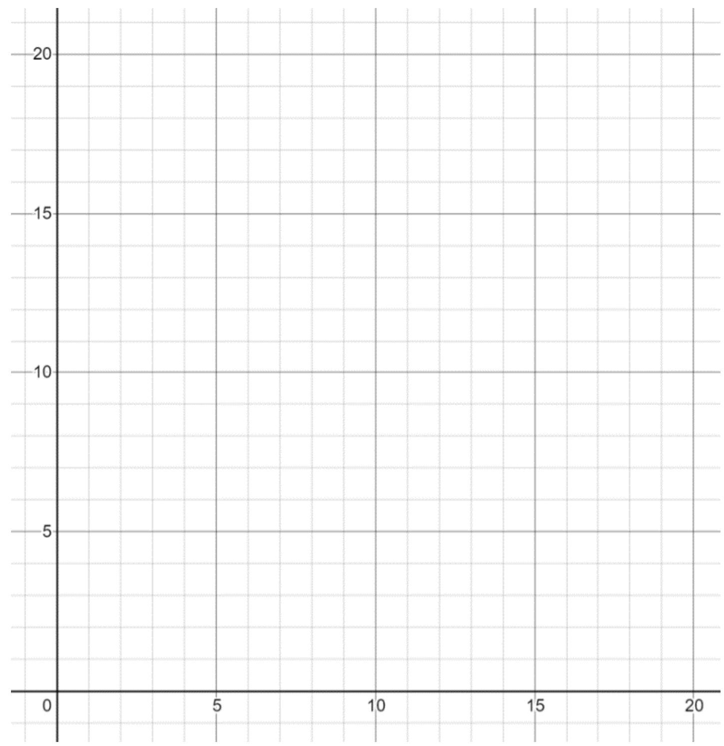
Try this question!

1) Determine the vertices of the polygon of constraints given the following scenario:

John loves his stuffed animals. His collection has stuffed bears and stuffed dinosaurs.

- John has a maximum of 20 stuffed animals.
- John knows he has at least 10 stuffed animals.
- John has less than or equal to 6 bears.
- John has at least 5 more dinosaurs than bears.

Given that each bear is worth $10 and each dinosaur is worth $5, what is the minimum value of John's collection and how many of each animal would John have if his collection was worth the minimum value?



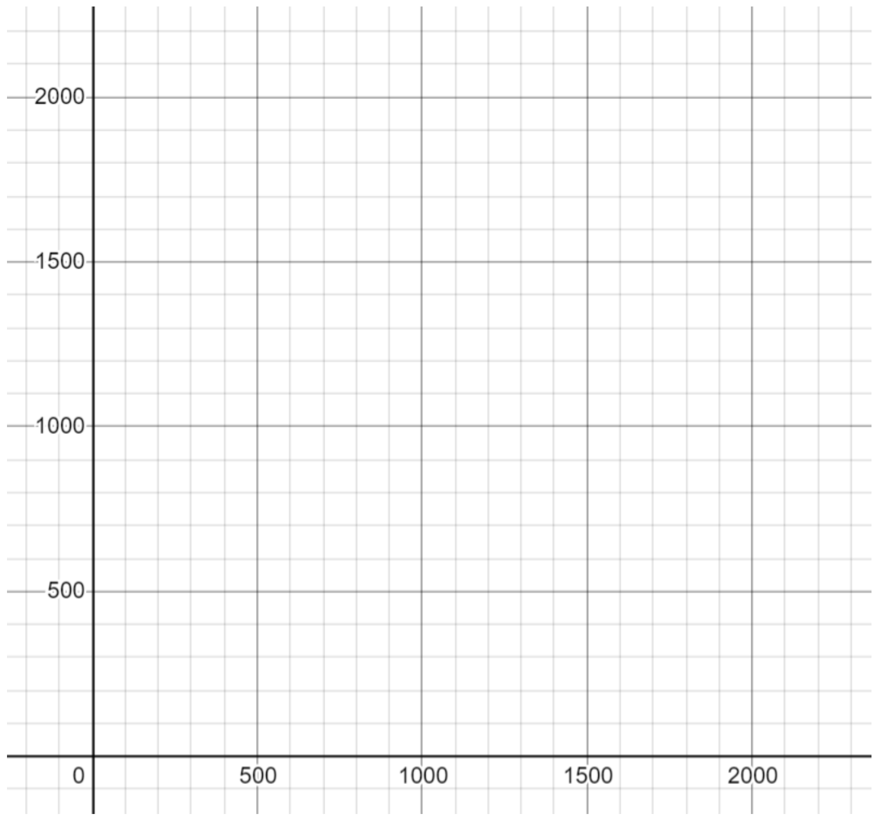
Optimization Unit – Target Objective, Optimizing Function, and Answering the Question

Optimization Unit – Target Objective, Optimizing Function, and Answering the Question

Practice Problems

1) A car manufacturer builds compact cars and minivans and wants to maximize its weekly profit. The profit generated from each car is $4000 and the profit generated from each minivan is $10 000.
   - The manufacturer's weekly production capacity is 2100 at most.
   - The manufacturer must build at least 1000 compact cars weekly.
   - The manufacturer must build at least 200 minivans weekly.
   - The number of compact cars built each week must be at least twice as many as the number of minivans built each week.

   What is the maximum profit the manufacturer can earn weekly?

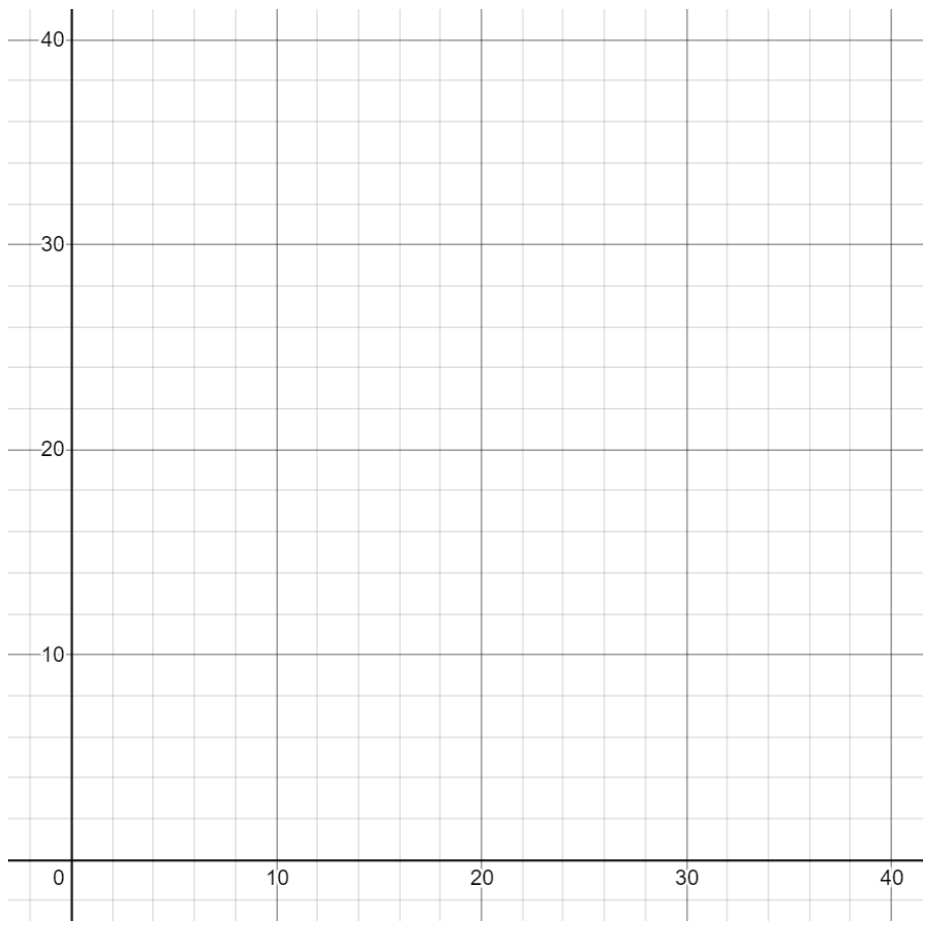


Optimization Unit – Target Objective, Optimizing Function, and Answering the Question

2) In order to treat a patient, a doctor decides to administer a treatment combining two medications, A and B. The side effects associated with these medications force the doctor to respect the following constraints:
   - The dosage of medication A must be at least 5 mg.
   - The dosage of medication A cannot exceed 15 mg.
   - The dosage of medication B must be at least 8 mg.
   - The dosage of medication B cannot exceed 25 mg.
   - The total dosage of medication cannot exceed 35 mg.

   The doctor wants the medications to be as effective as possible. Efficacy can be determined by using the following optimizing function: $Efficacy = 0.0305x + 0.025y$ where:
   - x is the amount of medication A
   - y is the amount of medication B

   What combination of medications should the doctor give in order to achieve the maximum efficacy?



Optimization Unit – Target Objective, Optimizing Function, and Answering the Question

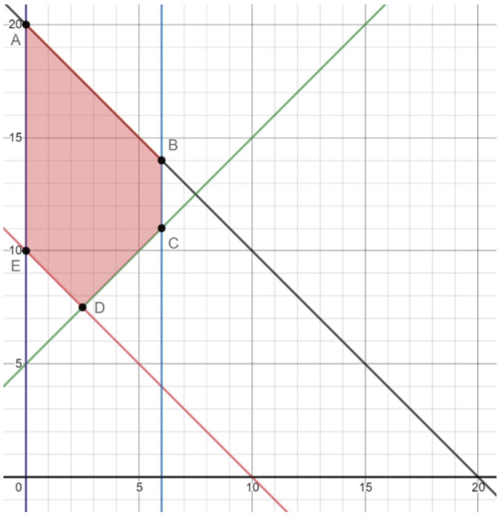
Optimization Unit – Target Objective, Optimizing Function, and Answering the Question
Answer Key

Questions in the Notes

1)

Let x be the number of stuffed bears

Let y be the number of stuffed dinosaurs



Optimizing Function: $Value = 10x + 5y$

Vertex A	**(0, 20)**	$\boldsymbol{Value = 100}$
Vertex B	**(6, 14)**	$\boldsymbol{Value = 130}$
Vertex C	**(6, 11)**	$\boldsymbol{Value = 115}$
Vertex D	**(2.5, 7.5)**	$\boldsymbol{Value = 62.5}$
Vertex E	**(0, 10)**	$\boldsymbol{Value = 50}$

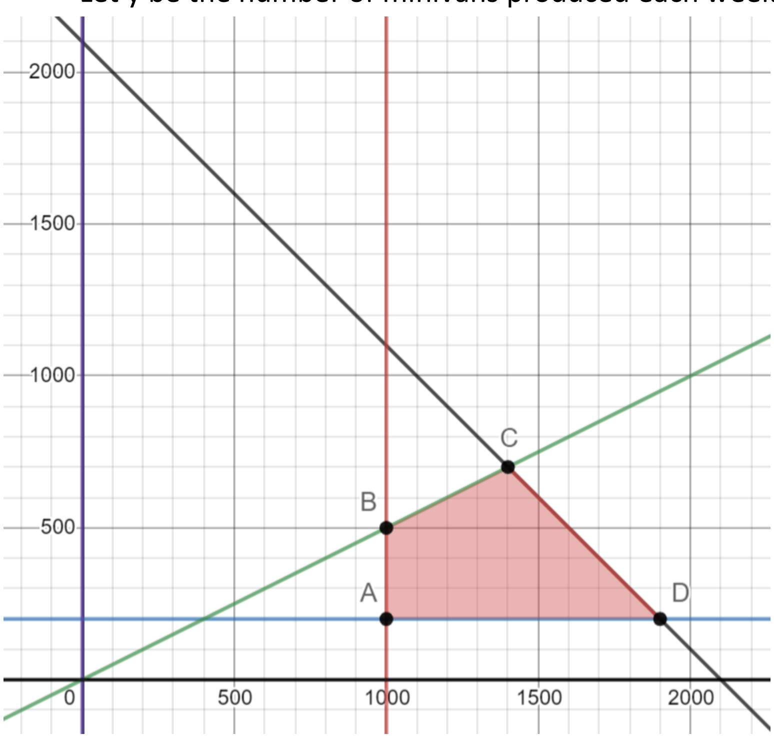
The minimum value of John's collection is $50 and if John's collection was worth the minimum, he would have 0 bears and 10 dinosaurs.

Practice Questions

1)

Let x be the number of compact cars produced each week

Let y be the number of minivans produced each week



$Profit\ = 4000x + 10000y$

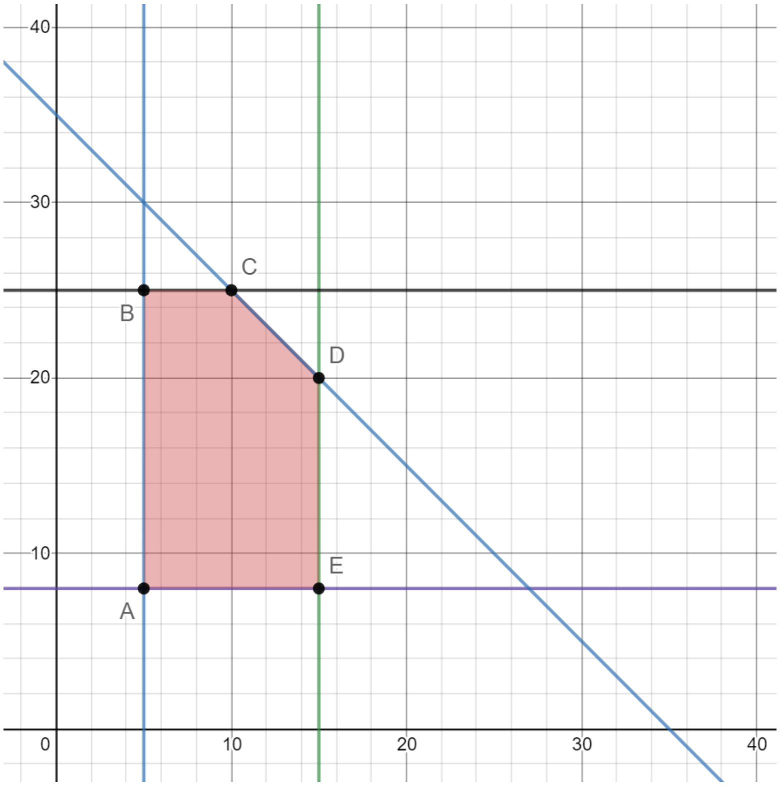
Vertex A	**(1000, 200)**	$\boldsymbol{P = 6\ 000\ 000}$
Vertex B	**(1000, 500)**	$\boldsymbol{P = 9000\ 000}$
Vertex C	**(700, 1400)**	$\boldsymbol{P = 16\ 800\ 000}$
Vertex D	**(1900, 200)**	$\boldsymbol{P = 9\ 600\ 000}$

The maximum profit the manufacturer can earn weekly is $16 800 000.

2)

Let x be the amount of medication A

Let y be the amount of medication B



$$Efficacy = 0.0305x + 0.025y$$

Vertex A	**(5, 8)**	$\mathbf{E = 0.3525}$
Vertex B	**(5, 25)**	$\mathbf{E = 0.7775}$
Vertex C	**(10, 25)**	$\mathbf{E = 0.93}$
Vertex D	**(15, 20)**	$\mathbf{E = 0.9575}$
Vertex E	**(15, 8)**	$\mathbf{E = 0.6575}$

The doctor should give 15 mg of medication A and 20 mg of medication B in order to achieve maximum efficacy.

2.8 OPTIMIZATION COMPLICATIONS: DECIMALS, DOTTED LINES, and TIES

There are three potential complications to optimization: decimals, dotted lines, and ties.

Decimals

In optimization, the vertices always need to be whole numbers (not decimals). This is because we are talking about things that can only be produced or sold (for example) in whole units.

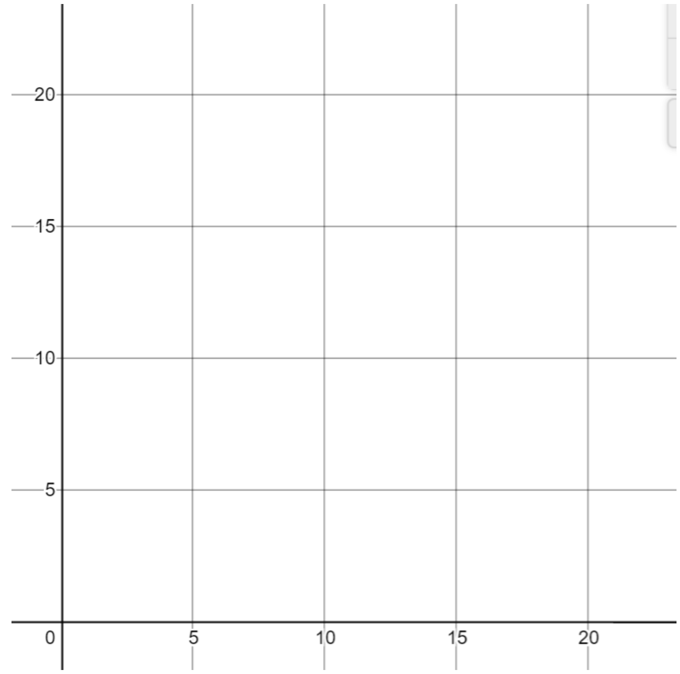
If a school is holding a car wash charges $5 to wash a car and $8 to wash a truck, it doesn't make sense to think about washing half a car. The students wouldn't get paid if they only washed half the car.

When we solve for the vertices, we always need to find a whole number (not a decimal). Sometimes we will get a decimal, though (think back to the example of John and his stuffed animals). When this happens, find a point near the vertex where x and y are both whole numbers either in the shaded area or on a solid line. Use that point instead of the decimals.

Ex: John loves his stuffed animals. His collection has stuffed bears and stuffed dinosaurs.

- John has a maximum of 20 stuffed animals.
- John knows he has at least 10 stuffed animals.
- John has less than or equal to 6 bears.
- John has at least 5 more dinosaurs than bears.

Given that each bear is worth $10 and each dinosaur is worth $5, what is the maximum value of John's collection?


20
15
10
5
0
5
10
15
20

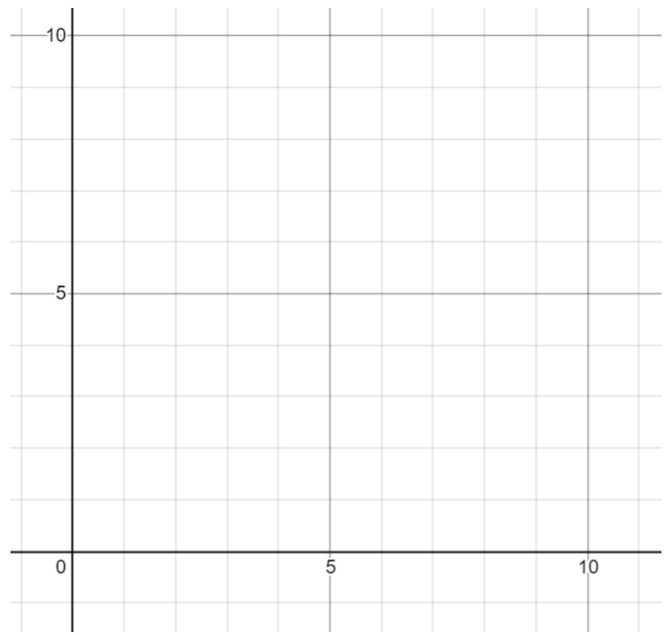
Optimization Unit – Complications: Decimals, Dotted Lines, and Ties

Try this question!

1) Reese has a collection of red and blue marbles.

- The collection contains at most 10 marbles.
- Reese has at least twice as many blue marbles as red marbles.
- Reese has at least 1 red marble.

Given that red marbles are worth \$2 and blue marbles are worth \$1.50, what is the minimum value of Reese's collection?



Optimization Unit – Complications: Decimals, Dotted Lines, and Ties

Dotted Lines

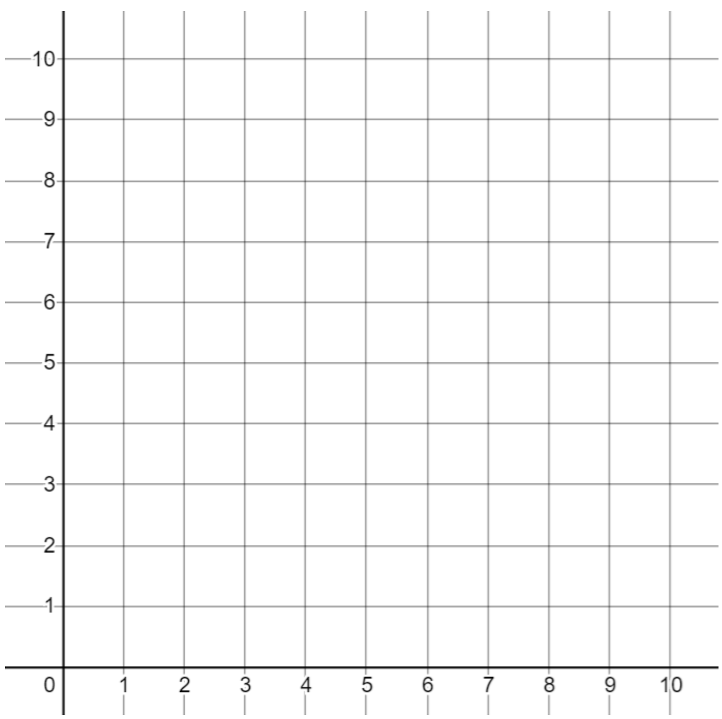
The second complication that can occur in optimization is when a vertex is on a dotted line (which means the inequality has a $>$ or $<$ instead of a $\geq$ or $\leq$.

This means the vertex itself is not included in the solution set, so cannot be the answer. However, a point close to the vertex could be the answer. We solve this problem just like we did for vertices that are decimals. Find a point near the vertex where x and y are both whole numbers and either in the shaded area or on a solid line. Use that point instead of the vertex that is on a dotted line.

Ex: A coffee shop sells tea and coffee according to the following constraints:

- Each hour, the shop must sell fewer than 10 beverages.
- Each hour, the shop sells at least 1 tea.
- Each hour, the shop sells at least as many coffees as teas.

The coffee shop earns a profit of $0.75 for each tea sold and $1.00 for each coffee sold. How many teas and coffees must the shop sell each hour in order to earn the maximum profit?



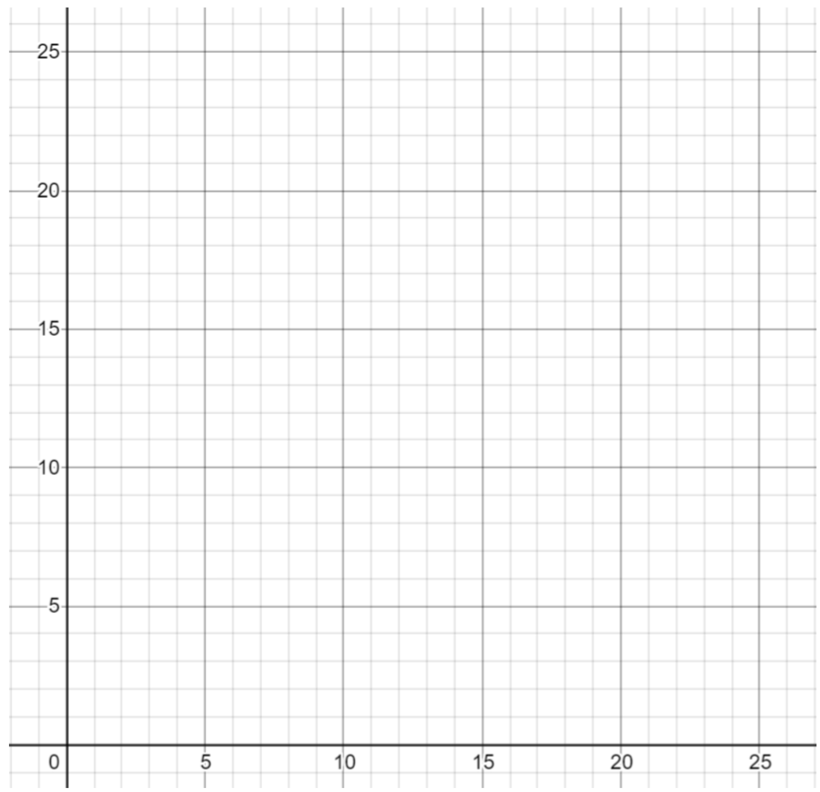
Optimization Unit – Complications: Decimals, Dotted Lines, and Ties

Try this question!

2) A pizza shop sells whole pizzas and slices of pizza.

- The shop must sell fewer than 15 whole pizzas per hour.
- The shop can sell no more than 25 items per hour.
- The shop sells a minimum of 5 slices of pizza per hour.
- The shop must sell at least 5 whole pizzas per hour.

Given that the shop earns a profit of $4 per whole pizza and $1.50 per slice of pizza, how many slices should the store per hour in order to earn the maximum profit?



Optimization Unit – Complications: Decimals, Dotted Lines, and Ties

Ties

The final complication present in some optimization questions is when two vertices tie for a maximum or minimum after using the optimizing function.

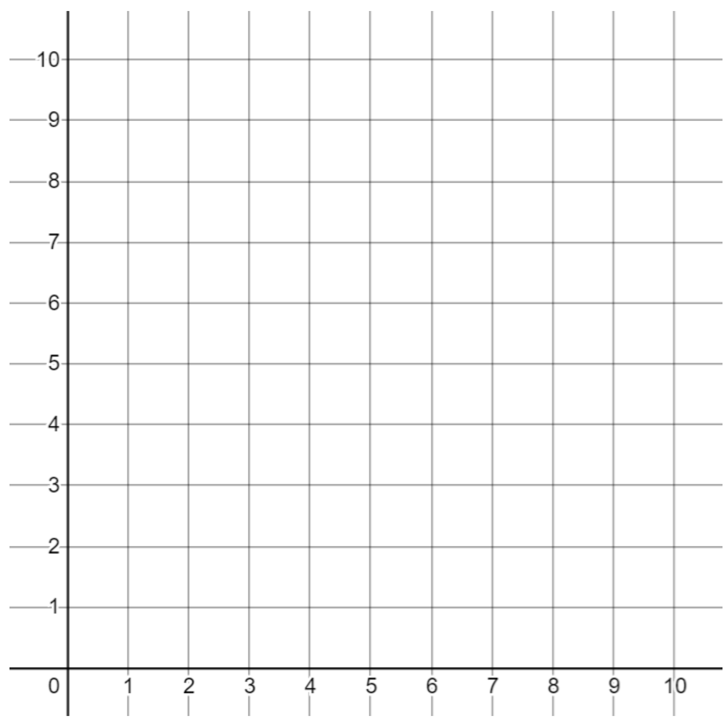
When two vertices tie for the optimal value, they are both solutions to the scenario. In addition, all the points on the line connecting the two tied vertices are also optimal solutions.

Ex: Students at Philemon Wright are selling scented candles as a fundraiser for their annual trip. They are selling lavender scented candles and vanilla scented candles.

- They sell at most 10 candles each day.
- They sell at least 3 lavender scented candles each day.
- They sell at least 2 vanilla scented candles each day.

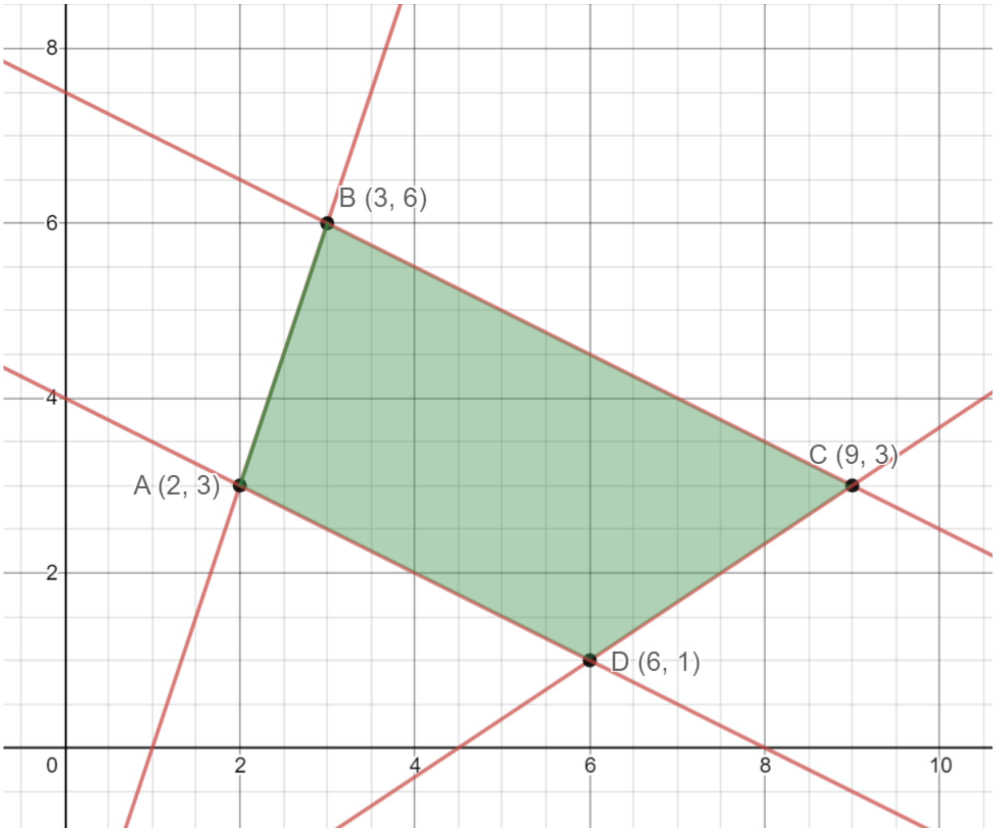
The profit for the lavender scented candles is $3 and the profit for the vanilla scented candles is also $3.

What is the maximum profit the students make each day and what are all the possible combinations of scents to maximize the value?



Try this question!

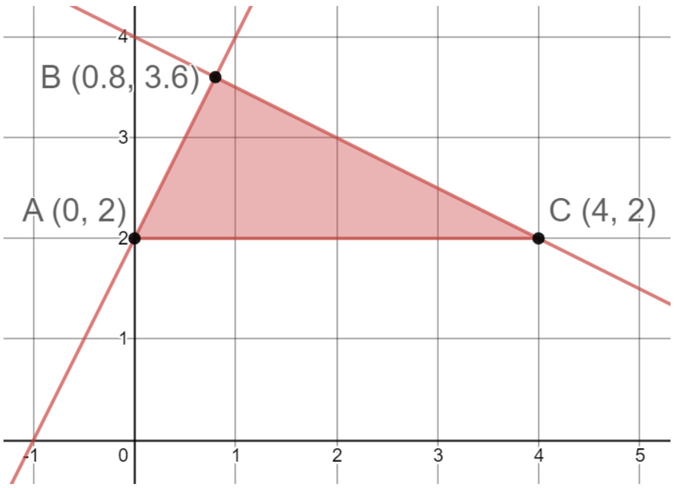
3) The polygon of constraints of an optimization question is represented on the graph below. The optimizing function is: $Profit = 3x + 6y$. How many points maximize this situation? What are they?



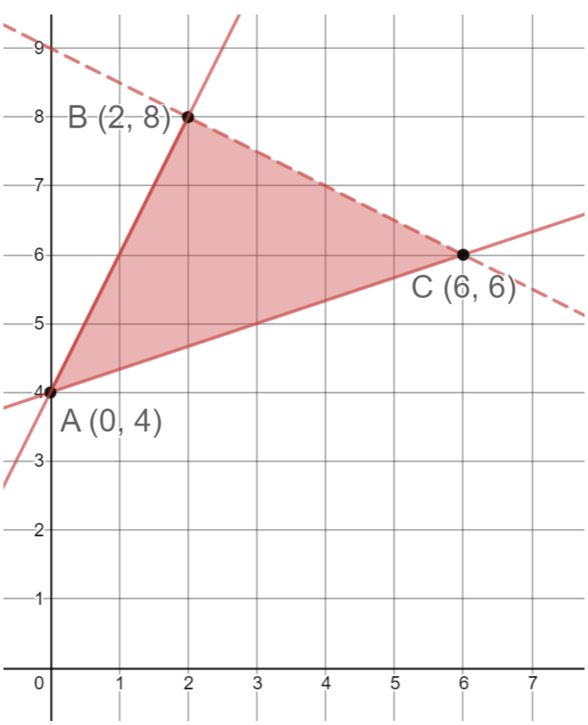
Optimization Unit – Complications: Decimals, Dotted Lines, and Ties

Practice Questions

1) The polygon of constraints of an optimization question is represented on the graph below. The optimizing function is: $Cost = \$4x + \$2y$. What is the minimum cost in this scenario?

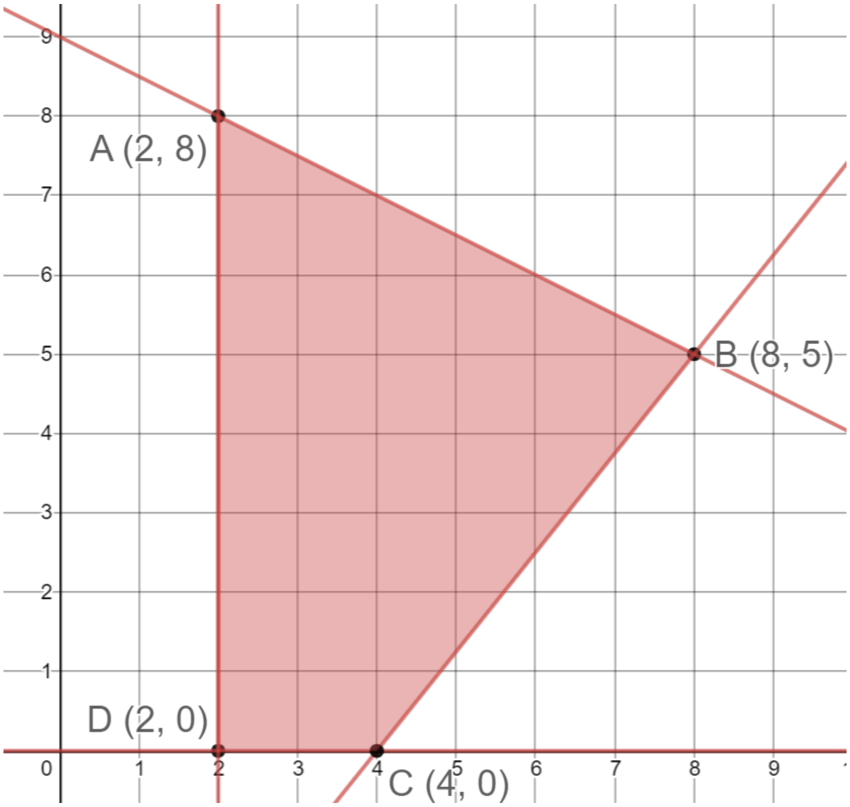


2) The polygon of constraints of an optimization question is represented on the graph below. The optimizing function is $Value = \$2x + \$10y$. What is the maximum value in this scenario?



Optimization Unit – Complications: Decimals, Dotted Lines, and Ties

3) The polygon of constraints of an optimization question is represented on the graph below. The optimizing function is $Value = 4x + 8$.
   What is the maximum value and how many points maximize the scenario?



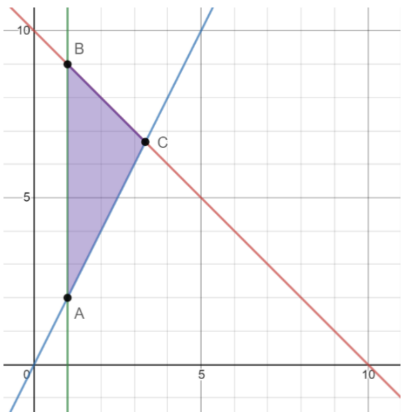
Optimization Unit – Complications: Decimals, Dotted Lines, and Ties
Answer Key

Questions in the Notes

1)

Let x be the number of red marbles

Let y be the number of blue marbles



: $Value = 2x + 1.5y$

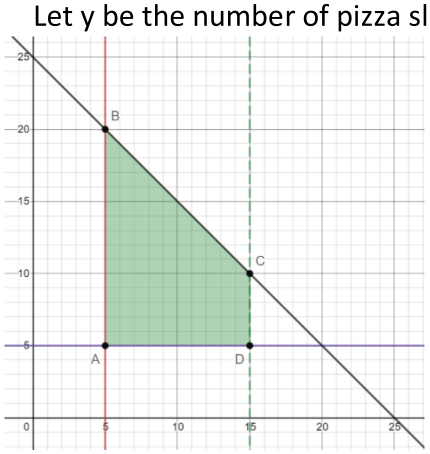
Vertex A	**(1, 2)**	$Value = 5$
Vertex B	**(1, 9)**	$Value = 15.5$
Vertex C	**(3.33, 6.66) change to (3, 7)**	$Value = 16.5$

The maximum value of Reese's collection is $16.50.

2)

Let x be the number of whole pizzas sold per hour

Let y be the number of pizza slices sold per hour



$$Profit = 4x + 1.5y$$

Vertex A	**(5, 5)**	$\boldsymbol{Profit = 27.5}$
Vertex B	**(5, 20)**	$\boldsymbol{Profit = 50}$
Vertex C	(15, 10) change to **(14, 11)**	$\boldsymbol{Profit = 72.5}$
Vertex D	(15, 5) change to **(14, 6)**	$\boldsymbol{Profit = 67.5}$

The pizza shop must sell 11 slices of pizza to earn the maximum profit.

3) $Profit = 3x + 6y$

Vertex A (2, 3)
$Profit = 24$

Vertex B (3, 6)
$Profit = 45$

Vertex C (9, 3)
$Profit = 45$

Vertex D (6, 1)
$Profit = 24$

Vertices B and C both tie for the maximum profit. Therefore, they are both solutions, as are all the points in between them.

There are 4 points that maximize the situation: (3, 6), (5, 5), (7, 4), and (9, 3).

Optimization Unit – Complications: Decimals, Dotted Lines, and Ties

Practice Questions

1)

Vertex A (0, 2)

$Cost = 4$

Vertex B (0.8, 3.6)
Change vertex to (1, 3)
$Cost = 10$

Vertex C (4, 2)

$Cost = 20$

Minimum cost is $4.

2)

Vertex A (0, 4)

$Value = 40$

Vertex B (2, 8)
Change to (2, 7)
$Value = 74$

Vertex C (6, 6)
Change to (5, 6)
$Value = 70$

Maximum Value is $74.

3)

Vertex A (2, 8)
$Value = 72$

Vertex B (8, 5)
$Value = 72$

Vertex C (4, 0)
$Value = 16$

Vertex D (2, 0)
$Value = 8$

Maximum Value is $72.
Both vertices A and B produce this maximum value, so all the points on the line connecting A and B are also solutions. There are 4 points that maximize the scenario.

2.9 OPTIMIZATION EXAM STYLE QUESTIONS

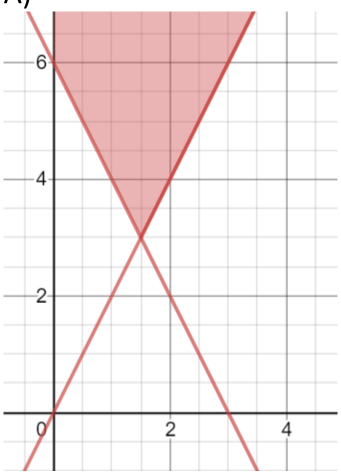
Multiple Choice

1) The system of inequalities below represents the constraints associated with an optimization situation.

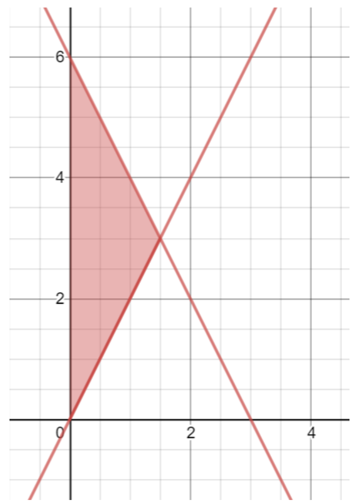
$$y \geq 0$$
$$x \geq 0$$
$$y \geq 2x$$
$$4x + 2y \leq 12$$

Which of the following represents the solutions for this system of inequalities?

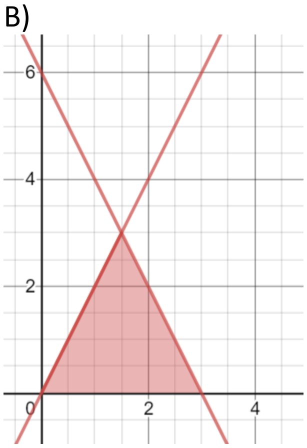
A)



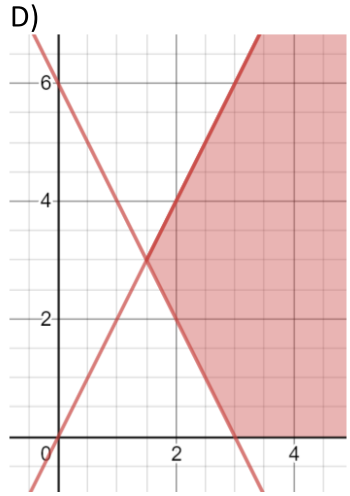
C



B)



D)



2) The graph below represents the polygon of constraints associated with an optimization situation.

Which of the following system of inequalities corresponds with this optimization system?

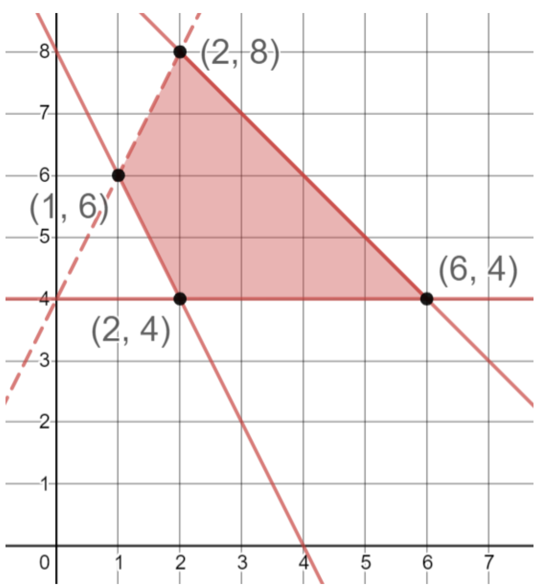
A)
$y \geq 0$
$x \geq 0$
$x \geq 3y$
$2x + 4y \leq 15$

B)
$y \geq 0$
$x \geq 0$
$x \leq 3y$
$2x + 4y \leq 15$

C)
$y \geq 0$
$x \geq 0$
$x \leq 3y$
$2x + 4y \geq 15$

D)
$y \geq 0$
$x \geq 0$
$x \geq 3y$
$2x + 4y \geq 15$

3) The polygon of constraints below is associated with an optimization situation.
The optimizing function is $Z = 100x + 100y$



How many points on the graph maximize the situation?

A) 1

B) 2

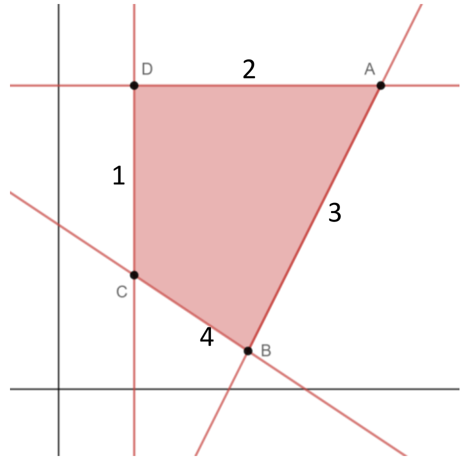
C) 4

D) 5

Short Answer

4) The constraints associated with an optimization situation are represented by the systems of inequalities and the polygon of constraints shown below. Each side of the polygon and its corresponding inequality are identified by the same number.

1) $x \geq 2$
2) $y \leq 8$
3) $y \geq 2x + 9$
4) $2x + 3y \geq 13$



What are the coordinates of vertex B of this polygon of constraints?

5) A high school is selling t-shirts and hoodies as a fundraiser for the class trip.
where x: the number of t-shirts sold
y: the number of hoodies sold

Translate the following statements into inequalities.

- **The school will sell a maximum of 200 items.**
- **There will be at least twice as many t-shirts as hoodies sold.**

6) Each year Grade 11 students hold a car wash as a fundraiser for Prom. The revenue raised from the car wash is represented by the optimizing function $Z = 8x + 10y$

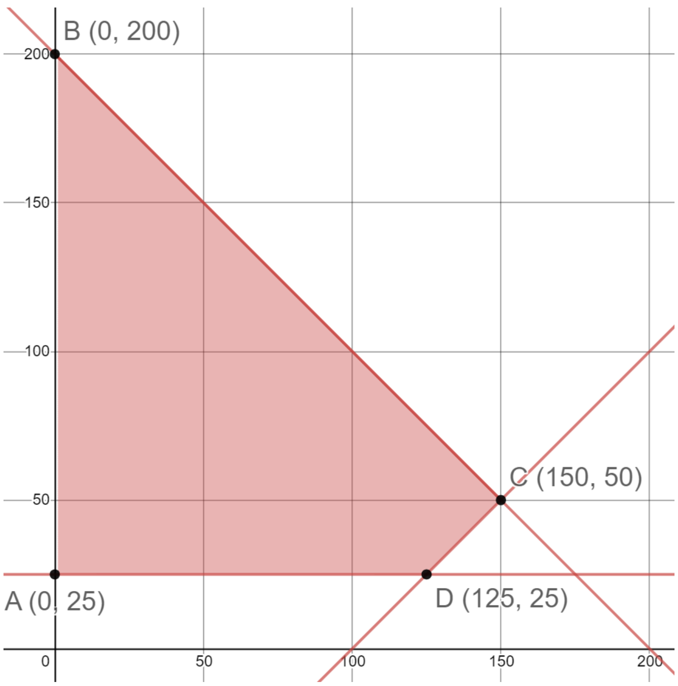
where: x is the number of cars washed

y is the number of trucks washed

The polygon of constraints below represents the combination of cars and trucks washed in a typical year.

This year, there is heavy construction near the car wash site. The organizers expected there will be fewer cars and trucks to was as a result. The decrease in vehicles is represented by the inequality below.

$$x + y \leq 150$$



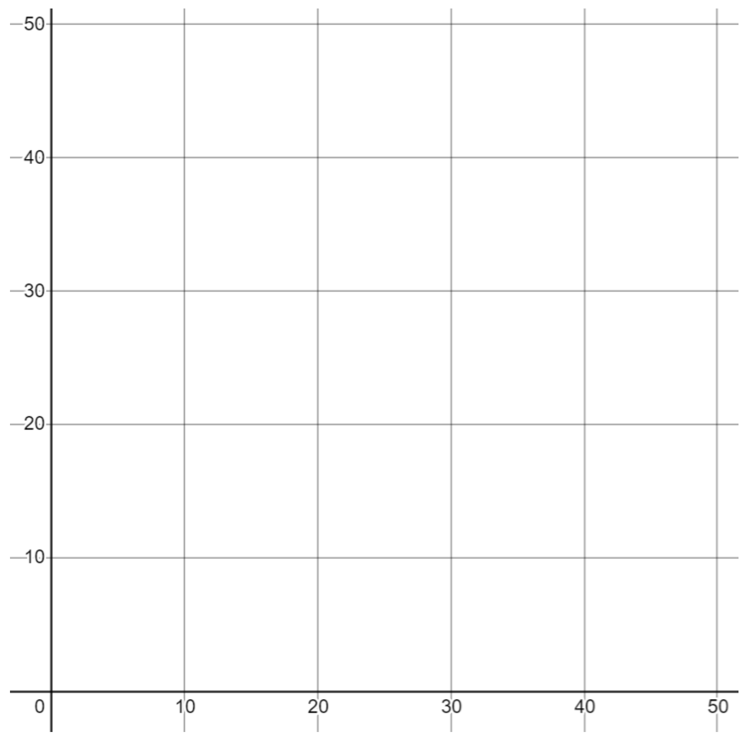
Vertex	Revenue $Z = 8x + 10y$
A (0, 25)	\$250
B (0, 200)	\$2000
C (150, 50)	\$1700
D (125, 25)	\$1250

By how much will the students' expected maximum revenue at the car wash decrease as a result of the construction?

Long Answer

7) Mackenzie is selling tulip bulbs as part of the school band fundraiser. They will offer a choice between standard red tulip bulbs and special orange tulip bulbs. Mackenzie's sales are limited by the following constraints:
   - Mackenzie can sell a total of 40 bulbs per day.
   - Mackenzie must sell a maximum of 10 red bulbs per day.
   - Mackenzie sells at least twice as many orange bulbs as red bulbs.

   Given that Mackenzie earns $1 for every red bulb sold and $2 for every orange bulb sold, what is the maximum profit Mackenzie can earn each day?



8) A company sells two different products. The first item is soap. The second item is lotion. Information about the sales of both items is below.

Soap

The company sells soap in two varieties: bars of soap and bottles of liquid soap. The soap sales must fit within the constraints given below:

- The company sells a maximum of 40 soaps per day.
- At least 5 bars of soap are sold each day.
- No fewer than 5 bottles of liquid soap are sold each day.
- The company sells no more than 3 times as many bars of soap as bottles of liquid soap

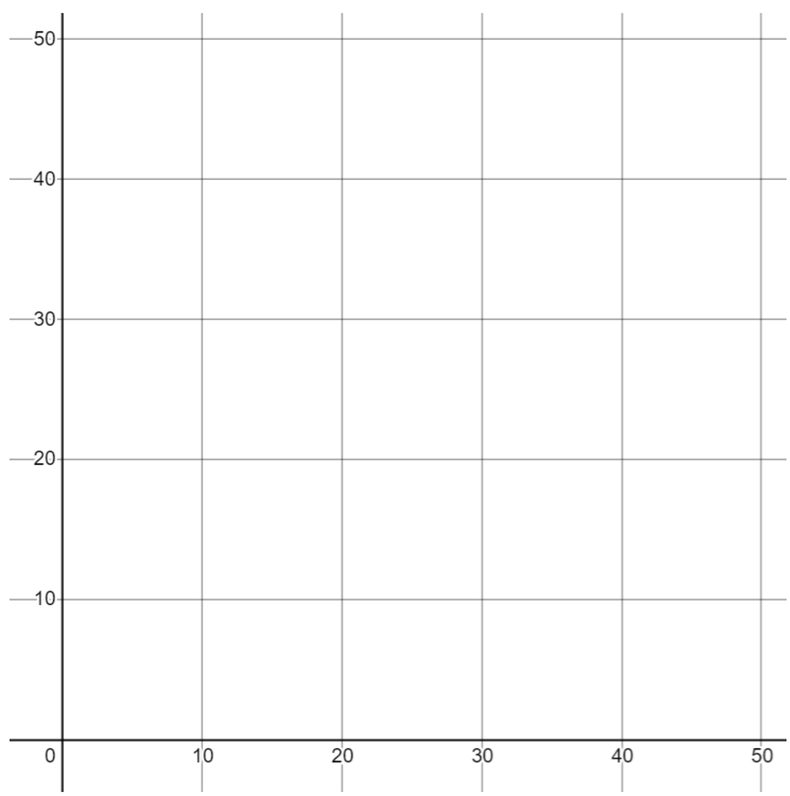
The company earns a profit of $2 per bar of soap and $3 per bottle of liquid soap.

Lotion

The company also sells two types of lotion: hand lotion and body lotion. Every day they sell 20 bottles of hand lotion and 25 bottles of body lotion.

The company earns the same profit on the sale of lotion as the maximum profit from the sale of soap.

Given that they earn a profit of $3 on each bottle of hand lotion, what is the profit earned on each bottle of liquid soap?



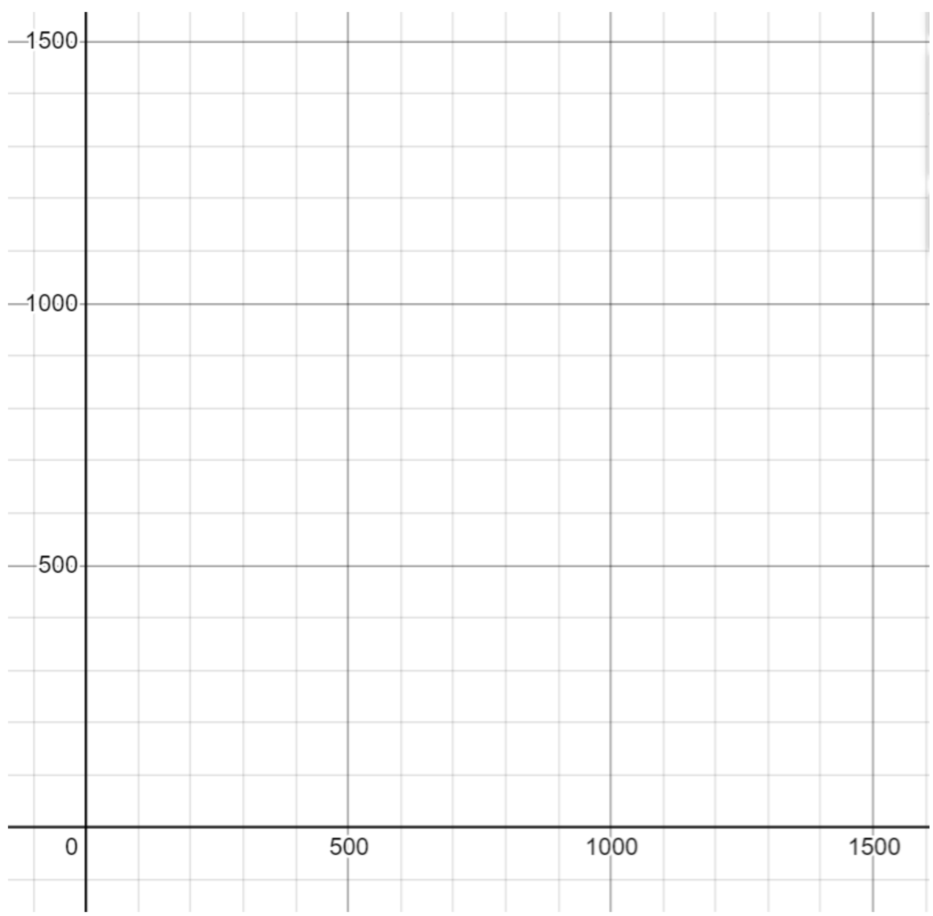
9) A company owns a vehicle manufacturing plant which has two assembly lines. One of the lines makes cars and one of the lines makes trucks. On a typical day, the plant must adhere to the following constraints in manufacturing cars and trucks.

- The plant can make no more than 1200 vehicles per day.
- The assembly line making cars can produce a minimum of 200 cars per day.
- The assembly line making trucks can produce no more than 800 cars per day.
- The manufacturing plant makes no more than twice times as many cars as trucks.

The company makes a profit of $2000 per car and $2500 per truck.

Today the manufacturing plant experienced a breakdown in the assembly lines and as a result the production capacity has been reduced. Today, the plant can make no more than 900 vehicles.

By how much did the breakdown reduce the maximum profit when compared to a typical day?



Review Unit – Linear Systems

Answer Key

1) **C**

2) **B**

3) **C**

4) **(5, 1)**

5) $x + y \leq 200$ $(or\ y \leq -x + 200)$ **and** $x \geq 2y$ $(or\ 2y \leq x\ or\ y \leq \frac{1}{2}x\ or\ y \leq 0.5x)$

6) **The expected maximum revenue will decrease by $500.**

7) **The maximum Mackenzie can earn each day is $80.**

8) **The profit earned on each bottle of liquid soap is $2.20.**

9) **$650 000**