Piecewise Functions

Piecewise Functions Basics

A piecewise function is made up of several functions, defined over different intervals of the domain, creating "pieces".

- The graph of a piecewise function is determined by the different functions that are elements of the piecewise function
- Domain: varies (look at the function to determine)
- Range: varies (look at a sketch to determine)

Sketching a Piecewise Function

Sketch each piece of the piecewise function over the stated domain (it helps to determine the y values of the minimum and maximum x value, as well as any asymptotes or vertices)

Ex: Sketch the following piecewise function and determine its domain and range.

$$f(x) = \begin{cases} 4x - 8 & -\infty \leq x \leq 2 \\ \sqrt{x-2} & 2 \leq x \leq 11 \\ \frac{4}{3}|14 - x| - 1 & 11 \leq x \leq 15 \end{cases}$$

$$\boldsymbol{y = 4x - 8}$$

This is a line with an initial value of -8 and a slope of 4.
The line begins at $x = -\infty$
The line must pass through the point $(0\ \ -8)$
The endpoint of the line is when $x = 2$. Solve for y when $x = 2$

$$y = 4(2) - 8$$
$$y = 0$$

The line ends at the point $(2, 0)$

$$\boldsymbol{y = \sqrt{x - 2}}$$

This is a square root function where a and b are both positive
The function begins at $x = 2$, so solve for y when $x = 2$

$$y = \sqrt{2 - 2}$$
$$y = 0$$

The function begins at $(2, 0)$
The function ends at $x = 11$ so solve for y when $x = 11$

$$y = \sqrt{11 - 2}$$
$$y = 3$$

The function ends at (11, 3)

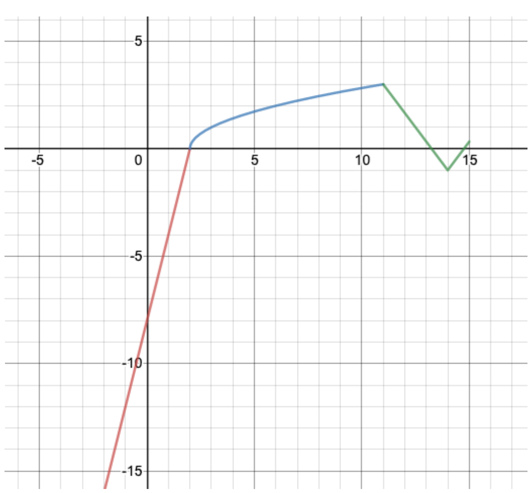
$$\boldsymbol{y = \frac{4}{3}|14 - x| - 1}$$

This is an absolute value function where a is positive
The function has a vertex at (14, -1) which is within the domain, so that is a point on the graph.
The function begins at $x = 11$ so solve for y when $x = 11$

$$y = \frac{4}{3}|14 - 11| - 1$$
$$y = 3$$

So the function begins at (11, 3)
The function ends at $x = 15$ so solve for y when $x = 15$

$$y = \frac{4}{3}|14 - 15| - 1$$
$$y = \frac{1}{3}$$



Domain: $]-\infty, 15]$

Range: $]-\infty, 3]$

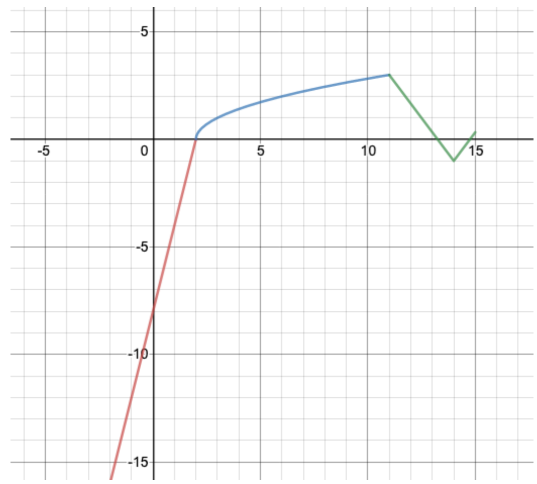
Solving Piecewise Functions

To solve for y given x, check the domain of each piece and use the correct rule.

To solve for x given y, solve using each piece of the function and then check the solution to see if the x value falls in the given domain for the piece used (a sketch can help).

Ex: Given the piecewise function below

$$f(x) = \begin{cases} 4x - 8 & -\infty \le x \le 2 \\ \sqrt{x-2} & 2 \le x < 11 \\ \frac{4}{3}|14 - x| - 1 & 11 \le x \le 15 \end{cases}$$



a) Solve for $f(8)$

We know $x = 8$ so we use $y = \sqrt{x-2}$

$$y = \sqrt{x-2}$$
$$y = \sqrt{8-2}$$
$$y = \sqrt{6} = 2.4495$$

b) Solve for x when $y = 3$

Using $y = 4x - 8$

$$y = 4x - 8$$
$$3 = 4x - 8$$
$$11 = 4x$$
$$\frac{11}{4} = x$$
$$x = 2.75$$

But 2.75 is not within the domain for this piece, so it is not a solution.

Using $y = \sqrt{x-2}$

$$y = \sqrt{x-2}$$
$$3 = \sqrt{x-2}$$
$$9 = x - 2$$
$$11 = x$$

But 11 is not within the domain for this piece, so it is not a solution.

Using $y = \frac{4}{3}|14 - x| - 1$

$$y = \frac{4}{3}|14 - x| - 1$$
$$3 = \frac{4}{3}|14 - x| - 1$$
$$4 = \frac{4}{3}|14 - x|$$
$$3 = |14 - x|$$
$$3 = 14 - x \text{ or } -3 = 14 - x$$
$$x = 11 \text{ or } x = 17$$

Only $x = 11$ is within the domain, so the solution is $x = 11$

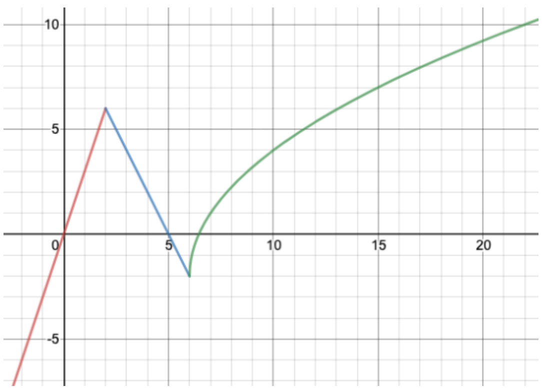
Solving Piecewise Inequalities

To solve an inequality: solve using each piece of the function and then check the solution to see if the x value falls in the given domain for the piece used (a sketch can help).

Ex: Given the piecewise function below, determine when $f(x) \geq 3$

$$f(x) = \begin{cases} 3x & -\infty \leq x \leq 2 \\ -2|x-2| + 6 & 2 \leq x \leq 6 \\ 3\sqrt{x-6} - 2 & 6 \leq x \leq \infty \end{cases}$$

Sketch the function:



$\boldsymbol{y = 3x}$

$3 = 3x$

$1 = x$

This is a solution.

$\boldsymbol{y = -2|x-2| + 6}$

$3 = -2|x-2| + 6$

$\frac{3}{2} = |x-2|$

$x - 2 = \frac{3}{2} \quad or \quad x - 2 = -\frac{3}{2}$

$x = \frac{7}{2} = 3.5 \quad or \quad x = \frac{1}{2} = 0.5$

This is a solution — This is not a solution

$\boldsymbol{y = 3\sqrt{x-6} - 2}$

$3 = 3\sqrt{x-6} - 2$

$\frac{5}{3} = \sqrt{x-6}$

$\frac{25}{9} = x - 6$

$x = \frac{79}{9} = 8.7778$

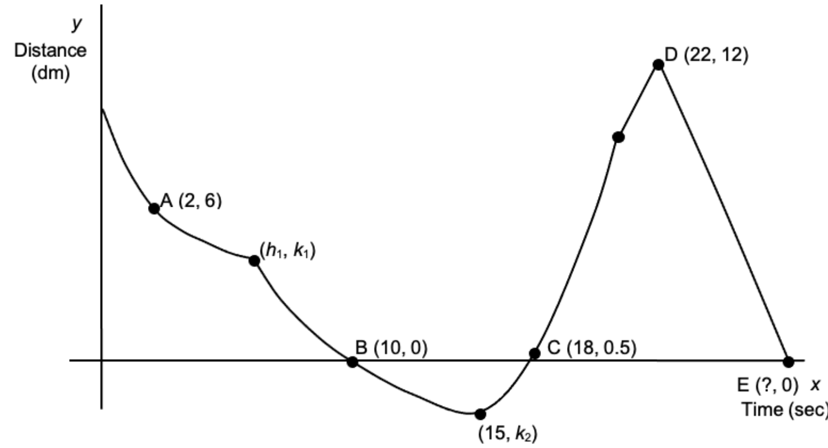
This is a solution

$$\therefore \boldsymbol{f(x) \geq 3 \ over \ \left[1, \frac{7}{2}\right] \cup \left[\frac{79}{9}, +\infty\right[}$$

Putting the Pieces Together

Calvin the chipmunk is being chased by his brother Isidore, according to the piecewise function shown below.

$$f(x) = \begin{cases} \dfrac{p}{x+2} + 2 & 0 \le x \le 6 \\ q\sqrt{x - h_1} + k_1 & 6 \le x \le 15 \\ r(x-15)^2 + k_2 & 15 \le x \le 21 \\ s|x - 22| + 12 & 21 \le x \le ? \end{cases}$$



Isidore catches Calvin at point E. How long does it take for Isidore to catch Calvin?

Step 1: Determine p (using point A)

$$y = \frac{p}{x+2} + 2$$

$$6 = \frac{p}{2+2} + 2$$

$$4 = \frac{p}{4}$$

$$p = 16$$

$$y = \frac{16}{x+2} + 2$$

Step 2: Determine (h_1, k_1), which is the vertex of the second function, so $h_1 = 6$

$$y = \frac{16}{x+2} + 2$$

$$y = \frac{16}{6+2} + 2$$

$$y = \frac{16}{8} + 2$$

$$y = 4$$

$$(h_1, k_1) = (6, 4)$$

Step 3: Determine q (using point B)

$$y = q\sqrt{x-6} + 4$$

$$0 = q\sqrt{10-6} + 4$$

$$-4 = 2q$$

$$q = -2$$

$$y = -2\sqrt{x-6} + 4$$

Step 4: Determine k_2

$$y = -2\sqrt{15-6} + 4$$

$$y = -2\sqrt{9} + 4$$

$$y = -2$$

Step 5: Determine r (using point C)

$$y = r(x-15)^2 - 2$$

$$0.5 = r(18-15)^2 - 2$$

$$2.5 = r(3)^2$$

$$r = \frac{2.5}{9} = \frac{5}{18}$$

Step 6: determine y when $x = 21$

$$y = \frac{5}{18}(x-15)^2 - 2$$

$$y = \frac{5}{18}(21-15)^2 - 2$$

$$y = \frac{5}{18}(6)^2 - 2$$

$$y = 8$$

So the unlabeled point is $(21, 8)$

Step 7:determine s

$$y = s|x-22| + 12$$

$$8 = s|21-22| + 12$$

$$-4 = s|-1|$$

$$-4 = s$$

Step 8: Solve for $y = 0$

$$0 = -4|x-22| + 12$$

$$3 = |x-22|$$

$$x - 22 = 3 \ \ or \ \ x - 22 = -3$$

$$x = 25 \ \ or \ \ x = 19$$

$X = 25$ is a solution ($x = 19$ is not)

$$\therefore It\ takes\ 25\ seconds\ for\ Isidore\ to\ catch\ Calvin$$