Unit 2: Graphs and Graph Theory
2.05 Special Paths and Circuits

Euler Path:

In a connected graph, a path that travels over each edge once and ONLY ONCE is called an **EULER PATH**
An **EULER PATH** exists when a graph contains exactly 2 vertices whose degrees are odd numbers.
An **EULER PATH** must start at a vertex having an odd-numbered degree and end at another vertex with an odd-numbered degree.

Ex: The Groomer

Gerald is in charge of maintaining the trails at a snowmobile club, and checks all of the trails each day. Here is a graph showing the trails that lead to rest areas.

To save time and money, he wants to groome all of the trails without covering the same trail twice.
Can it be done? Yes
Is there more than one way to do it? Yes
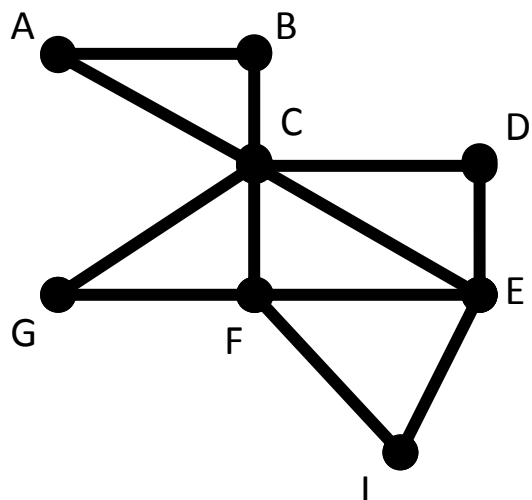What are all the possible starting points? Lakeside or Cliffside
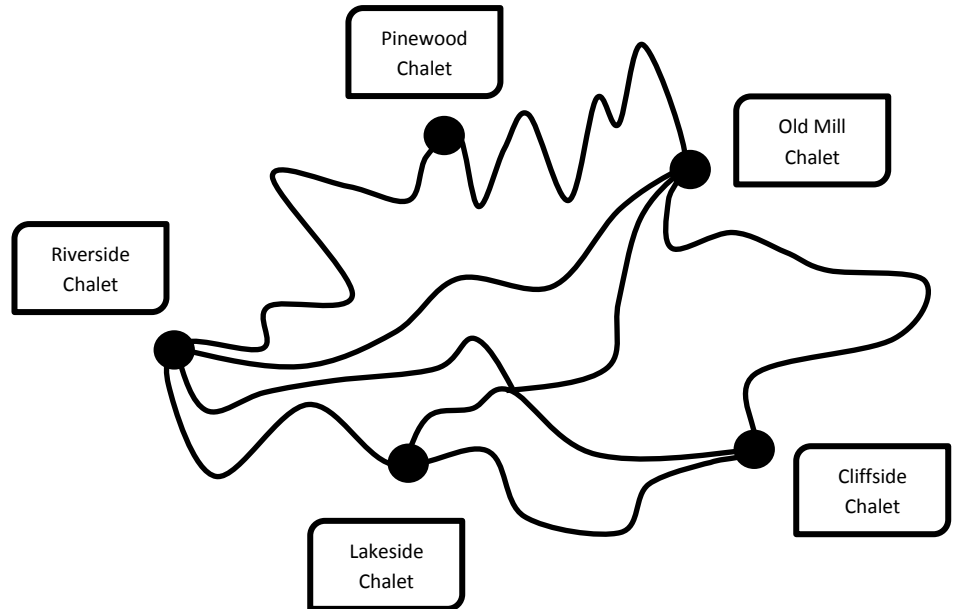
Euler Circuit:

An Euler Path that begins and ends at the same vertex is called an **EULER CIRCUIT**
An **EULER CIRCUIT** exists when the degrees of all vertices are even numbers
An **EULER CIRCUIT** can begin at any vertex and ends at the same vertex.

Ex:

Does this graph contain an EULER CIRCUIT?

Yes, every vertex has an even degree.

You can get to every vertex using a path that crosses every edge once and only once.

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          Visions   Page 27 (EULER) #2, 5cd, 6, 7, 8 (HAM) #4, 5b, 12, 14,   (BOTH) #3, 15, 16, 17, 18, 19
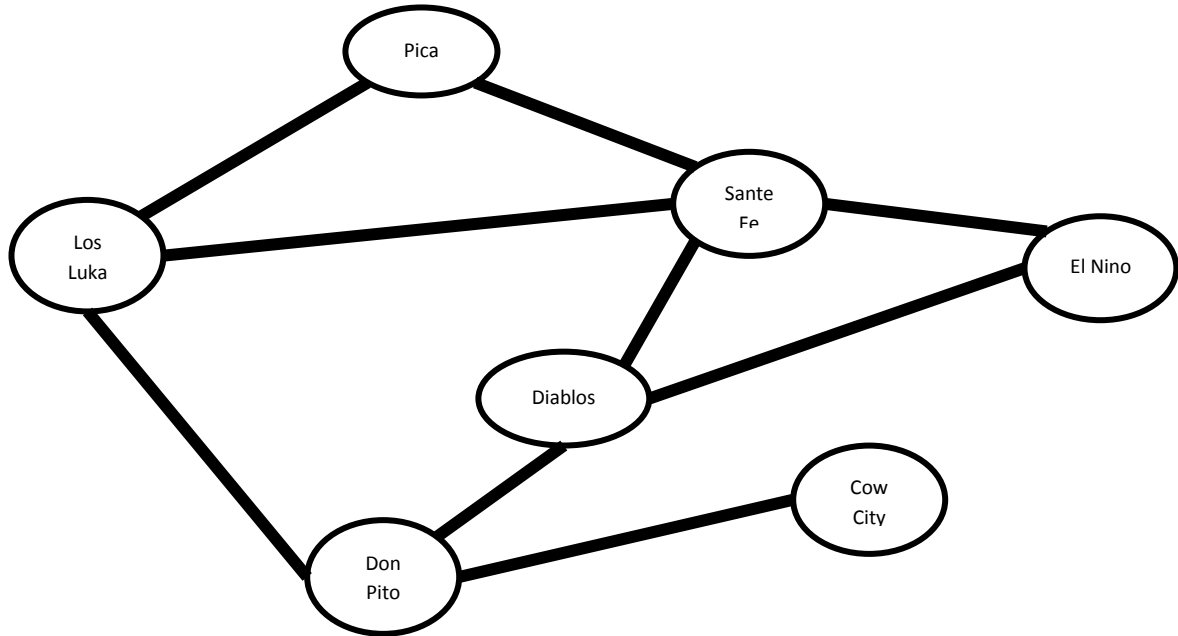
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Hamiltonian Path:

In a connected graph, a path that passes through every vertex once and only once is called a
HAMILTONIAN PATH

Ex: The Wild West

Billy the Kid is on a major streak. One of the area deputies has taken it upon himself to plot Billy's
activities. Below is a map of the area:

Knowing his chances of getting caught are greater if he goes through the same city more than once, Billy
never looks back.

Deputy Dan has noted the following:

- In the last 5 days Billy robbed banks in Pica, Diablos, and Los Luka
- Billy has already hit 5 cities
- He has yet to be less than 2 lengths from Cow City.

Using this information: Which city is next on Billy's list?

<p style="text-align:center; color:red">Don Pito</p>

With the above information in mind, label the Hamiltonian Path that Billy used if he was able to dodge
Deputy Dan throughout the area

<p style="text-align:center; color:red">EDSPLD$_p$C   or   LPSEDD$_p$C</p>

Hamiltonian Circuit:

A Hamiltonian Path that begins and ends at the same vertex is called a **HAMILTONIAN CIRCUIT**
A graph in which each vertex is joined to at least half of the other vertices contains a **HAMILTONIAN
CIRCUIT**

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