Unit 2: Graphs and Graph Theory
2.09 Path of Optimal Value

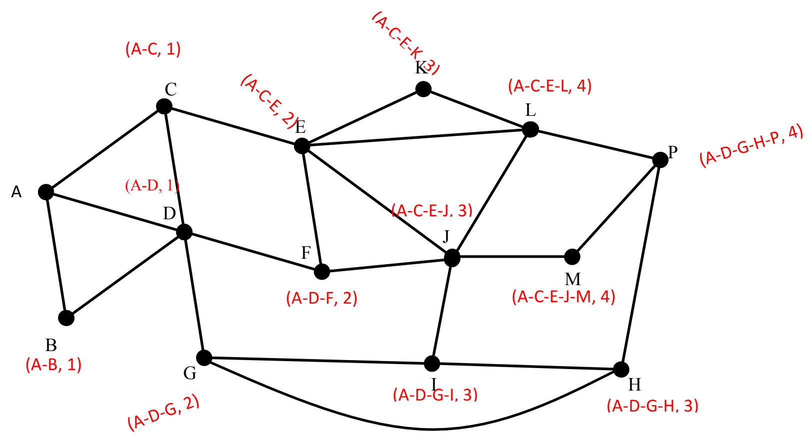
On a graph the **PATH OF MINIMUM VALUE** corresponds to the **DISTANCE** (shortest path) between two vertices

On a graph the **PATH OF MAXIMUM VALUE** corresponds to the **SIMPLE PATH** (no repeated edges) that travels the maximum number of edges

On a weighted graph (network) the **PATH OF OPTIMAL VALUE** corresponds to the **WEIGHT** of the path between two vertices (either max. or min. depending on the question)

You can determine the path of minimum value connecting two vertices in a graph by indicating, at each vertex, the path of minimum value to that point from the initial vertex.

Example: Find the Path of Minimum Value



$\therefore$ *ADGHP is the Path of minimum value at 4 units*

When dealing with a basic connected graph, it can be argued that it's just as easy to count the edges, however, once weights are applied to the edges, the process of labelling each vertex on the graph with the distance becomes more helpful.

Practice: Page 52+ # 4, 6, 8, 9, 10

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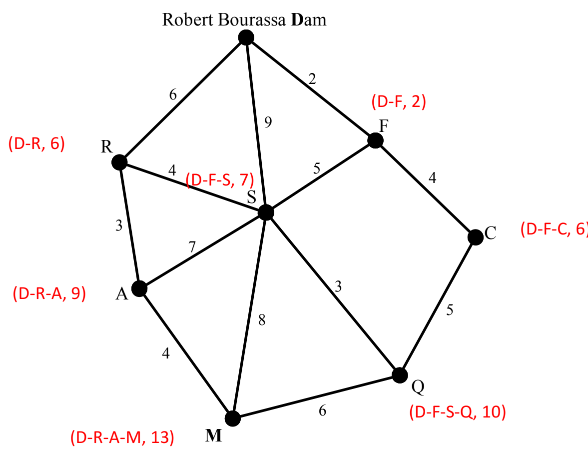
Example: Robert Bourassa Hydroelectric Development

The La Grande 2 powerhouse, dam and reservoir – named the Robert Bourassa Hydroelectric Development in 1996 – is located on the La Grande River, east of James Bay. The facility generates approximately 5328 MW of electricity, which is transmitted via high-tension lines to Quebec's main urban centres. These transmission lines lose some of their power depending on their state of repair and other factors. The graph below shows some of these lines and the power loss in tens of megawatts over a given period. The facility's engineering crew has to find the path of least power loss from the Dam to M at any given time.



$\therefore DRAM \text{ is the Path of minimum value at } 13\ MW$

Practice: Page 52+ # 4, 6, 8, 9, 10

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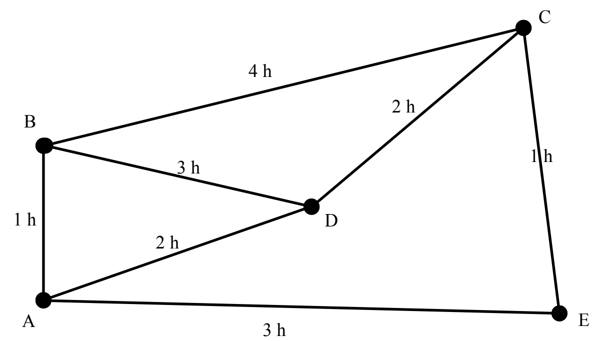
Example: Outdoor Medicine

A doctor must visit several villages in the Far North. He lives in village E and travels by snowmobile. The graph below shows the territories he must visit. He usually spends 1 h in each village.



a) Give a path that doctor can follow to visit all the villages without passing through the same village twice, and without returning home after his trip. Is there more than one solution? Hamiltonian Path
- ECDBA
- ECDAB
- EABDC
- EADBC
- ECBDA
- ECBAD
- EABCD
- EADCB

b) Give a path the doctor can follow to visit all the villages without passing through the same village twice, but which brings him home at the end of the trip. Is there more than one solution? Hamiltonian Circuit
- ECBDAE = 17 h
- ECDBAE = 14 h

c) Which path should he follow if he wants to get home as early as possible on the same day
- **ECDBAE**

d) Why is it pointless to look for the least value of an Euler path or circuit?
- Sum of every edge doesn't change ☺

Practice: Page 52+ # 4, 6, 8, 9, 10