Voting Unit – Plurality
4.1 PLURALITY

There are many different ways to count votes in order to determine a winner.

- Canada, the United States, and many countries in Africa use **plurality**.
- The Australian House of Representative, the Indian presidential election, and parliament in Papau New Guinea use **majority with elimination**.
- Nauru, a tiny island country in Micronesia (just northeast of Australia) uses **Borda count**.
- The **Condorcet method** is not practical to use in elections, but a modified version of it is the basis for voting in Robert's Rules of Order.
- Most countries in western Europe and South America, as well as Russia, Kazakhstan, and several countries in Africa use **proportional representation**.

Each method has advantages and disadvantages.

Plurality

In voting under a plurality method, people vote for their favorite option. The option with the most votes is the winner.

Advantages: it is easy to calculate and a winner is guaranteed (unless there is a tie)

Disadvantages: A winner could emerge that most people do not want

Ex: Grade 11 students are electing their class president. Of the 150 students in grade 11, 70 vote for Ali, 30 vote for Brenda, and 50 vote for Chris. Under plurality, who wins the election?

In some questions, we will be given a ranked ballot where people put all the options in order.

Ex: Dr. James is going to bring a snack (apples, granola bars, or pepperoni sticks) in for her math students and the students will vote to determine the snack. The following table summarizes the results of the student vote.

# of votes / Preference	45	32	28	23
1st choice	apples	granola bars	pepperoni	granola bars
2nd choice	granola bars	apples	apples	pepperoni
3rd choice	pepperoni	pepperoni	granola bars	apples

Given these results, which snack would win under plurality?

Apples: 45

Granola Bars: 32 + 23 = 55

Pepperoni: 28

Granola Bars wins

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Voting Unit – Plurality
Practice Questions

1) In a hockey league, a committee of 53 members must select a recipient for the trophy for the hardest-working player from a list of three candidates.

- Candidate A received 16 votes
- Candidate B received 20 votes
- Candidate C received 17 votes

Under plurality voting, which candidate wins?

2) To determine Monday's menu at a high school cafeteria, the school's 250 grade 11 students were asked to rank three menus in order of preference. The results are presented in the table below.

# of votes Preference	63	51	46	45	24	21
1st choice	Pizza	Hamburgers	Tacos	Tacos	Hamburgers	Pizza
2nd choice	Hamburgers	Pizza	Pizza	Hamburgers	Tacos	Tacos
3rd choice	Tacos	Tacos	Hamburgers	Pizza	Pizza	Hamburgers

Under plurality voting, which menu item would be served?

3) A new school is being built and an election was held to determine if it will be in Village A, Village B, or Village C. Village A received 45% of the vote. Village B received 35% of the vote. Village C received 20% of the vote. Under plurality rule, where will the school be built?

Voting Unit - Majority
4.2 MAJORITY

In voting under a majority method, people vote for their favorite option. The option with more than half the votes is the winner.

Step 1: determine the total number of votes

Step 2: divide the total number of votes by 2

Step 3: determine the number of first place votes for each option

Step 4: determine a winner (the option with **more than half** the votes).

Note: if no option has more than half the votes, there is no winner.

Advantages: more than half the people are guaranteed to like the winner

Disadvantages: there may be no winner

Ex: A group of friends is trying to decide where to eat dinner. They decide to vote for their favorite type of food, and the results are presented in the table below. Which option wins using majority?

# of votes / Preference	4	4	7	5
1st choice	Vietnamese	Italian	Vietnamese	Sushi
2nd choice	Sushi	Vietnamese	Italian	Italian
3rd choice	Italian	Sushi	Sushi	Vietnamese

TOTAL VOTES

4+4+7+5 = 20

20÷2 = 10

Need more than 10 votes

Vietnamese: 4+7 = 11

Italian: 4

Sushi: 5

Vietnamese wins

Ex: A group of people are voting on their favorite winter activity. 70 vote for skiing, 30 vote for snowshoeing, and 50 vote for hockey. Under majority, which option wins?

TOTAL VOTES

70+30+50 = 150

150÷2 = 75

NEED MORE THAN 75 VOTES

SKIING: 70

SNOWSHOEING: 30

HOCKEY: 50

NO OPTION HAS MORE THAN 75 VOTES SO NO WINNER

1) The 40 members of the board of directors of a company must vote to elect the president of the board. Three candidates apply for the position and the results of the vote are presented in the table below.

# of votes / Preference	16	14	10
1st choice	A	C	B
2nd choice	B	A	A
3rd choice	C	B	C

Given the board of directors use majority to determine the winner, which candidate wins?

2) Grade 11 students are voting on where to go on their class trip. The results are presented in the table below.

# of votes / Preference	63	51	46	45	24	21
1st choice	Montreal	Montreal	Vancouver	Vancouver	Vancouver	Montreal
2nd choice	Halifax	Halifax	Montreal	Halifax	Halifax	Vancouver
3rd choice	Vancouver	Vancouver	Halifax	Montreal	Montreal	Halifax

Under majority voting, which location would win?

Voting Unit – Majority with Elimination
4.3 MAJORITY WITH ELIMINATION

In voting under a **majority with elimination** method, people rank the options from favorite to least favorite. The option with more than half the votes is the winner. However, if no option receives more than half the votes, the option with the fewest votes is eliminated and re-assigned.

Step 1: determine the total number of votes

Step 2: divide the total number of votes by 2

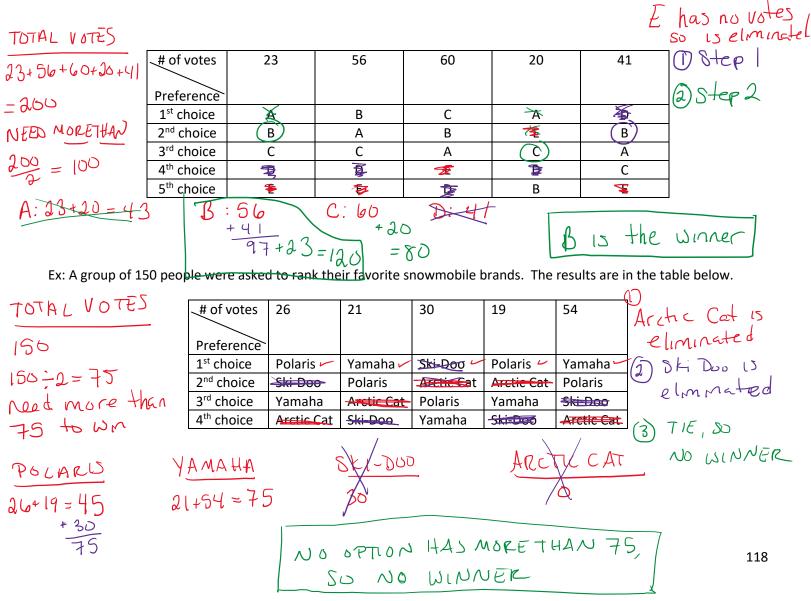Step 3: determine the number of first place votes for each option

Step 4: determine a winner (the option with **more than half** the votes)

Step 5: If no winner, the option with the fewest number of votes is eliminated and the votes are re-assigned to the next option on the list(s). Repeat this step until an option has more than half the votes

Advantages: A winner is guaranteed and people's preferences for all options are included

Disadvantages: Voting and counting the votes can be time-consuming depending on the number of options

Ex: A political party is holding elections to determine its new leader. There are 5 people nominated for the position.

Handwritten annotations (left):
TOTAL VOTES
$23+56+60+20+41$
$=200$
NEED MORETHAN
$\frac{200}{2}=100$

A: $23+20=43$ B: 56   C: 60   D: 41

B: $56+41 = 97 +23 = 120$

C: $60 +20 = 80$

# of votes	23	56	60	20	41
Preference					
1st choice	A	B	C	A	E
2nd choice	B	A	B	E	B
3rd choice	C	C	A	C	A
4th choice	D	D	E	D	C
5th choice	E	E	D	B	D

Handwritten annotations (right):
E has no votes so is eliminated
① Step 1
② Step 2
B is the winner

Ex: A group of 150 people were asked to rank their favorite snowmobile brands. The results are in the table below.

Handwritten annotations (left):
TOTAL VOTES
150
$150 \div 2 = 75$
Need more than 75 to win

POLARIS $26+19=45$ $+30$ $\overline{75}$

YAMAHA $21+54=75$

SKI-DOO 30

ARCTIC CAT 0

# of votes	26	21	30	19	54
Preference					
1st choice	Polaris	Yamaha	Ski-Doo	Polaris	Yamaha
2nd choice	Ski-Doo	Polaris	Arctic Cat	Arctic Cat	Polaris
3rd choice	Yamaha	Arctic Cat	Polaris	Yamaha	Ski-Doo
4th choice	Arctic Cat	Ski-Doo	Yamaha	Ski-Doo	Arctic Cat

Handwritten annotations (right):
① Arctic Cat is eliminated
② Ski Doo is eliminated
③ TIE, so NO WINNER

NO OPTION HAS MORE THAN 75, SO NO WINNER

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Voting Unit – Majority with Elimination

Practice Questions

1) The table below presents the results of an election between 3 candidates: A, B, and C.

# of votes / Preference	50	30	27	24	10
1st choice	C	B	A	A	B
2nd choice	B	C	B	C	A
3rd choice	A	A	C	B	C

Which candidate wins using majority with elimination?

2) Philemon Wright is selling grad hoodies, but can only order 1 color of hoodies. Students are asked to vote among the following options: Green, Blue, Red, and Black. The results of the vote are presented in the table below:

# of votes / Preference	52	41	35	33	29	21
1st choice	Blue	Green	Green	Black	Blue	Red
2nd choice	Green	Blue	Black	Blue	Green	Green
3rd choice	Black	Red	Red	Red	Red	Black
4th choice	Red	Black	Blue	Green	Black	Blue

Using majority with elimination, which color hoodie will be ordered?

Voting Unit – Borda Count
4.4 BORDA COUNT

In voting under a **Borda count** method, people rank the options from favorite to least favorite. Points are assigned to each choice. For each option, the number of votes is multiplied by the points, and all are added together. The option with the most points is the winner.

Step 1: Assign points to each option (last choice is 0, next is 1, next is 2, and so on until all choices are assigned)

Step 2: Every time an option is ranked, the number of votes is multiplied by the points assigned

Step 3: All the points for each option are added together

Step 4: The option with the most votes is the winner.

Advantages: This allows for a nuanced interpretation of preferences and will generally lead to a high degree of satisfaction among the electorate

Disadvantages: It's a complicated system to implement

Ex: A group of people are asked about what kind of movies they like best. They are given 3 options to rank: comedy, horror, and documentary. The results are shown in the table below.

# of votes / Preference	45	32	28	23
1st choice	Comedy *90*	Documentary *64*	Horror *56*	Documentary *46*
2nd choice	Documentary *45*	Comedy *32*	Comedy *28*	Horror *23*
3rd choice	Horror *0*	Horror *0*	Documentary *0*	Comedy *0*

x 2 *x 1* *x 0*

Given these results, which move type is most popular using Borda count?

Comedy
90 + 32 + 28 + 0
= 150

DOCUMENTARY
45 + 64 + 0 + 46
= 155

HORROR
0 + 0 + 56 + 23
= 79

Documentary has the most points so is the winner

Ex: A town is electing a new mayor. There are 4 candidates (A, B, C, and D). The results are presented in the table below.

CORRECTION

# of votes / Preference	300	450	430	380
1st choice	A *900*	C *1350*	D *1290*	D *1140*
2nd choice	C *600*	B *900*	B *860*	B *760*
3rd choice	B *300*	A *450*	C *430*	A *380*
4th choice	D *0*	D *0*	A *0*	C *0*

x 3 *x 2* *x 1* *x 0*

A
900 + 450 + 0 + 380
= 1730

B
300 + 900 + 860 + 760
= 2820

C
600 + 1350 + 430 + 0
= 2380

D
0 + 0 + 1290 + 1140
= 2430

B has the most points so is the winner

Voting Unit – Borda Count
Practice Questions

1) A group of friends is trying to decide how to spend a Saturday. They take a vote and the results are presented in the table below. Which option wins using Borda count?

# of votes / Preference	4	4	7	5
1st choice	Movie	Video Games	Movie	Museum
2nd choice	Museum	Movie	Video Games	Video Games
3rd choice	Video Games	Museum	Museum	Movie

2) Grade 11 students were asked to rank the following 3 classes from favorite to least favorite: Math, English, and French. The results are presented in the table below. Which option wins using Borda count?

# of votes / Preference	63	51	46	45	24	21
1st choice	Math	Math	English	English	French	Math
2nd choice	English	French	Math	Math	English	French
3rd choice	French	English	French	French	Math	English

Voting Unit – Condorcet Method
4.5 CONDORCET METHOD

The Condorcet method is a voting system in which the winner is the option that, when compared in a head-to-head competition with every other method, is the preferred option.

Step 1: Compare option A to option B. Determine the number of people who prefer A to B and the number of people who prefer B to A. The option with the most votes is the winner.

Step 2: Compare option A to another option and repeat step 1. Do this for as many options as exist.

Step 3: Compare option B to every other option, one option at a time (as in step 1).

Step 4: Repeat with every possible combination of options.

Step 5: Declare the winner – the option that never loses.

Advantages: this method is the most rigorous and assures the most number of people will be satisfied by the outcome.

Disadvantages: there is often no winner and it is complicated to implement.

Ex: The 40 members of the board of directors of a company must vote to elect the president of the board. Three candidates apply for the position and the results of the vote are presented in the table below.
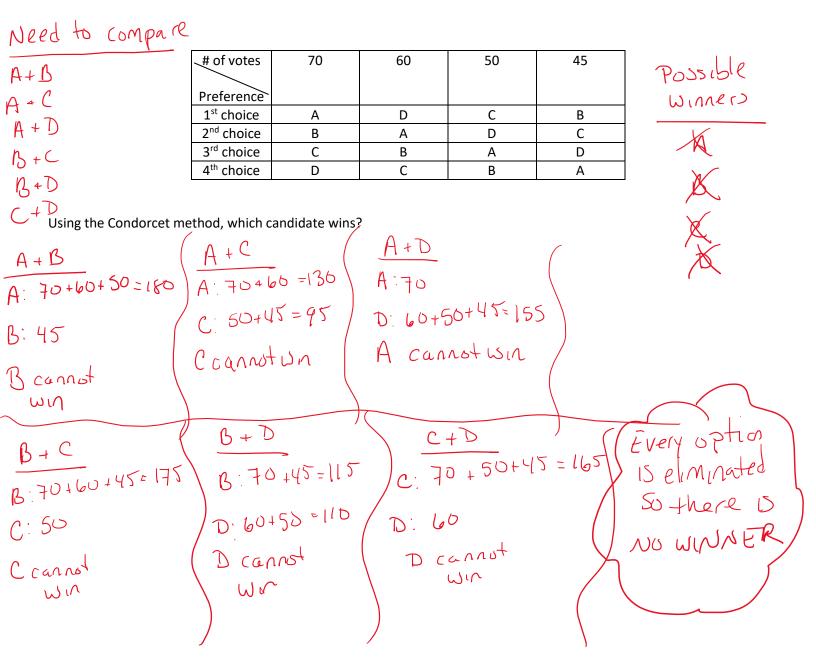
Need to Compare

A + B

A + C

B + C

# of votes	16	14	10
Preference			
1st choice	A	C	B
2nd choice	B	A	A
3rd choice	C	B	C

Given the board of directors use the Condorcet method to determine the winner, which candidate wins?

A + B

* A: 16 + 14 = 30

B: 10

B cannot be the winner

A + C

A: 16 + 10 = 26

C: 14

C cannot be the winner

B + C

B: 16 + 10 = 26

C: 14

C cannot be the winner.

Possible Winners

A

~~B~~

~~C~~

The only option not eliminated is A, so A is the winner

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Voting Unit – Condorcet Method

Ex: Voters are asked to choose between 4 candidates: A, B, C, and D. The results of the vote are presented in the table below:

Need to compare

A + B
A + C
A + D
B + C
B + D
C + D

# of votes / Preference	70	60	50	45
1st choice	A	D	C	B
2nd choice	B	A	D	C
3rd choice	C	B	A	D
4th choice	D	C	B	A

Possible Winners

~~A~~
~~B~~
~~C~~
~~D~~

Using the Condorcet method, which candidate wins?

A + B
A: 70+60+50 = 180
B: 45
B cannot win

A + C
A: 70+60 = 130
C: 50+45 = 95
C cannot win

A + D
A: 70
D: 60+50+45 = 155
A cannot win

B + C
B: 70+60+45 = 175
C: 50
C cannot win

B + D
B: 70+45 = 115
D: 60+50 = 110
D cannot win

C + D
C: 70+50+45 = 165
D: 60
D cannot win

Every option is eliminated so there is NO WINNER

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Voting Unit – Condorcet Method
Practice Questions

1) Philemon Wright is selling grad hoodies, but can only order 1 color of hoodies. Students are asked to vote among the following options: Green, Blue, Red, and Black. The results of the vote are presented in the table below:

# of votes / Preference	52	41	35	33	29	21
1st choice	Blue	Green	Green	Black	Blue	Red
2nd choice	Green	Blue	Black	Blue	Green	Green
3rd choice	Black	Red	Red	Red	Red	Black
4th choice	Red	Black	Blue	Green	Black	Blue

Using the Condorcet method, which color hoodie will be ordered?

2) A small group of people are asked their favorite fast-food restaurant. The results are presented in the table below.

# of votes / Preference	4	6	5
1st choice	McDonalds	Taco Bell	Pizza Pizza
2nd choice	Taco Bell	Pizza Pizza	McDonalds
3rd choice	Pizza Pizza	McDonalds	Taco Bell

Using the Condorcet method, which option is the most preferred?

Voting Unit – Proportional Representation
4.6 PROPORTIONAL REPRESENTATION

In **proportional representation**, individuals vote for the party they most prefer. The total number of votes for each party are calculated, and then the parties are awarded a number of seats in proportion to the number of votes received.

Step 1: Determine the number of votes for each party

Step 2: Determine the total number of votes

Step 3: Determine the proportion of votes for each party $\frac{votes\ for\ the\ party}{total\ number\ of\ votes}$

Step 4: Multiply the proportion of votes for each party by the number of seats available

Step 5: Ignore the decimals (do not round) and assign each party that number of seats

Step 6: Assign any remaining seats by choosing the highest decimal, the next highest, etc until all seats are assigned

Advantages: This system is quite representative of individual preferences and leads to a distribution of power that fairly accurately reflects the will of the electorate

Disadvantages: There is often no majority parts and thus requires a coalition government that can slow the decision making process.

Ex: A country is electing representatives to fill 90 seats in its Parliament. There are 5 parties in the election: A, B, C, D, E. The results of the election are presented in the table below.

Party	Number of Votes	PROPORTION	x # SEATS	# SEATS	Additnd seats	TOTAL
Party A	113	0.2128	19.152	19		19
Party B	108	0.2034	18.306	18		18
Party C	132	0.2486	22.374	22		22
Party D	86	0.1620	14.58	14	+1	15
Party E	92	0.1733	15.597	15	+1	16

TOTAL   531

TOTAL 88

need 2 more to get to 90

Using proportional representation, how many seats will each party earn?

Party A: 19 seats
Party B: 18 seats
Party C: 22 seats
Party D: 15 seats
Party E: 16 seats

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Voting Unit – Proportional Representation

Ex: A town is holding an election to fill (12) seats on its council. There are 3 parties and the results of the election are in the table below:

Party	Percent of Vote Received	x 12	# Seats	Add	TOTAL
A	38% ÷100 = 0.38	4.56	4	1	5
B	33.6% ÷100 = 0.336	4.032	4		4
C	28.4% ÷100 = 0.284	3.408	3		3

TOTAL: 11

Using proportional representation, how will the available seats be divided?

need 1 more

Party A: 5 seats

Party B: 4 seats

Party C: 3 seats

Voting Unit – Proportional Representation
Practice Questions

1) In 2019, Canada held a federal election and the number of popular votes each party received is in the table below.

Party	# of Votes
Liberal	6 018 728
Conservative	6 239 227
Bloc Quebecois	1 387 030
New Democratic Party	2 903 722
Green Party	1 189 607
Independent	72 546

a) If Canada used a proportional representation system, how many of the 338 seats in Parliament would each party receive?

b) The actual number of seats won by each party under our current system (districts using plurality) is in the table below. How would proportional representation change these results?

Party	# of Seats
Liberal	157
Conservative	121
Bloc Quebecois	32
New Democratic Party	24
Green Party	3
Independent	1

Voting Unit – Approval

4.7 APPROVAL

The final method of voting we will consider is Approval voting.

In approval voting, instead of ranking candidates, voters will select as many options as they like. Every option they choose will receive 1 vote. The option with the most votes is the winner.

A: 45+28+23 = 96 *

B: 32+28 = 60

Ex: The results of an election are presented in the table below.

C: 32+28 = 60

Number of Voters	45	32	28	23
	A	B	A	A
	D	C	B	
		D	C	

D: 45+32 = 77

Using approval voting, which option would win?

A is the winner

Practice Questions

1) A group of friends is trying to decide where to eat dinner. They decide to vote for their favorite type of food, and the results are presented in the table below. Which option wins using Approval voting?

# of votes	4	4	7	5
	Vietnamese Sushi	Italian Vietnamese	Vietnamese Italian Sushi	Sushi

2) To determine Monday's menu at a high school cafeteria, the school's 250 grade 11 students were asked to rank three menus in order of preference. The results are presented in the table below.

# of votes	63	51	46	45	24	21
	Pizza Hamburgers	Pizza Tacos	Tacos	Tacos Hamburgers	Hamburgers	Pizza

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Voting Unit – Exam Style Questions
4.8 EXAM STYLE QUESTIONS

Long Answer

1) The Grade 11 class at your high school is planning the graduation trip. Three possible destinations are offered to the students: New York, Boston and Chicago. The three trips are all comparable in price. All 241 grade 11 students were asked to give their order of preferences as to which destination they would like to visit. The results were recorded in the table below.

# Votes / Ranking	86	40	30	85
1st	New York	Boston	Boston	Chicago
2nd	Boston	Chicago	New York	Boston
3rd	Chicago	New York	Chicago	New York

The school administration agreed to choose the winning destination after analysing the results of the four following voting procedures: Borda, Elimination, Plurality and Condorcet.

Which destination was chosen by the school administration?

2) The grad committee will be selling hoodies to the 240 graduating students.

Three colours have been proposed for the hoodies: black, red, and green.

The grad committee asked the students to rank these colours in order of preference. The results of the vote are shown in the table below.

NUMBER OF VOTES	$\frac{1}{3}$ of the students	10 % of the students	?	15% of the students
1ST CHOICE	black	red	green	black
2ND CHOICE	red	green	red	green
3RD CHOICE	green	black	black	red

After analyzing the results of the vote, the students were told that the chosen colour was green.

Pamela, Erin, Brandon and Charlie made the following affirmations.

- Pamela thinks that the colour was chosen using plurality voting.
- Erin thinks that the colour was chosen using the elimination method.
- Brandon thinks that the colour was chosen using Borda count.
- Charlie thinks that the colour was chosen using the Condorcet method.

Among Pamela, Erin, Brandon and Charlie, whose affirmation is correct? Select all that apply.

Voting Unit – Exam Style Questions

3) Students were asked to rank 3 course options according to their preferences.

All 930 students in the school answered the survey. The results are indicated in the table below.

Number of students who ranked the course options in this way	20% of student population _____ students	$\frac{1}{3}$ of the student population _____ students	434 students
1st choice	Psychology	Biology	Media Studies
2nd choice	Biology	Media Studies	Biology
3rd choice	Media Studies	Psychology	Psychology

The teachers analyzed the data collected from the students using: Plurality voting, Borda count, Elimination method and Condorcet method.

The teachers made the following prediction: in three out of the four methods, the same course option will be ranked as their first choice.

Are the teachers correct in their prediction? Justify each of the methods.