# Optimization Unit – Definition and Steps

## 1.1 OPTIMIZATION DEFINITION AND STEPS

Mathematics can help us figure out optimal solutions.  Linear optimization helps us determine how to achieve maximum profit or minimum cost (for example).

Optimization is a long process, but we'll take it one step at a time.

Steps for Optimization:

1. Define variables ("Let x be… Let y be…")
2. Turn words into inequalities
3. Rearrange into $y = ax + b$ form
4. Graph and shade all inequalities
5. Identify polygon of constraints
6. Find (x, y) of each vertex (corner) of the polygon of constraints
7. Write optimizing function
8. Use optimizing function at each vertex
9. Choose maximum or minimum (depending on the question)
10. Write concluding sentence

## 1.2 OPTIMIZATION STEP 1

**Step 1**

The first step of optimization is defining our variable. To do this, we will read through the question and figure out the two things we're talking about.

Once we know the two things we care about, we will write two statements:

- "Let x be…"
- "Let y be…"

*Note: I typically use x and y as the variables. This will make it easier when we start graphing.*
*I also usually let x be the first variable mentioned in the question and y be the second variable mentioned, but the final answer won't change if you swap them.*

Ex: Define the variables for the following statement. There are red and blue marbles in a jar.

Let x be the number of red marbles

Let y be the number of blue marbles

**Practice Questions**

1) Define the variables in each of the scenarios below.

a) A pencil case contains pens and pencils.

b) Students are holding a car wash where they wash cars and trucks.

c) A garden grows red tulips and white tulips.

**1.3 OPTIMIZATION STEP 2**

Now that we've defined our variables, that will be the first thing we do in every optimization question. The next step is to turn statements into inequalities.

First, we'll look at turning statements into equalities.

Option 1: We are given a total amount. We will write the equation: $x + y = total$

Ex: Define the variables and turn the statement into an equation.
A bag contains red and blue marbles. There are a total of 15 marbles in the bag.

Let x be the number of red marbles

Let y be the number of blue marbles

$x + y = 15$

**Practice Questions**

1) Write an equation for each of the following scenarios

a) A garden grows roses and tulips. There are a total of 300 plants in the garden.

b) Students are holding a bake sale. They sell cookies and cupcakes. A total of 250 treats are sold.

c) John collects stuffed dinosaurs and stuffed bears. He has a total of 25 stuffed animals in his collection.

Optimization Unit – Step 2

Option 2: We are given a comparison statement. When we write the equation, x will be on one side and y on the other.

Hint: When reading comparison statements, define variables. See which variable comes first in the comparison statement. Write that variable on the left and the other variable on the right. Then figure out which thing you have more of. The multiplication (or addition) will go with the OTHER variable.

Ex: Define the variables and turn the statement into an equation.

A school is selling strawberry plants and tomato plants as a fundraiser. They sell twice as many strawberry plants as tomato plans.

Let $x$ be the number of strawberry plants

Let $y$ be the number of tomato plants.

$$x = 2y$$

Ex: Define the variables and turn the statement into an equation.

Students are holding a fundraiser washing cars and trucks. They wash 10 more trucks than cars.

Let $x$ be the number of cars

Let $y$ be the number of trucks $\quad y = x + 10$

**Practice Questions**

2) Write an equation for each of the following scenarios

a) A company sells road bikes and mountain bikes. They sell three times as many road bikes as mountain bikes.

b) Dr. James drinks tea and coffee. In any given week, she drinks 3 more teas than coffees.

c) A farm has goats and chickens. There are 5 times as many chickens as goats

Optimization Unit – Step 2

To change statements into inequalities, we will follow the same steps as changing statements to equalities, but instead of $=$ we will use $>, \geq, <,$ or $\leq$.

| Math Inequality Symbols and Words | |
|---|---|
| $<$ | **Less Than**<br>Is under<br>Is fewer |
| $>$ | **Greater Than**<br>Is more than<br>Is greater<br>Exceeds |
| $\leq$ | **Less Than or Equal To**<br>Is at most<br>Has a maximum of<br>Is not greater than<br>Does not exceed (go over)<br>Is not more than |
| $\geq$ | **Greater Than or Equal To**<br>Is at least<br>Is not less than<br>Is not under<br>Has a minimum value of |

When we are converting words to inequalities, this chart can help us determine which inequality to use.

Hint: when turning statements into inequalities, make sure you write the variables in the same order as they appear in the statement.

Ex: Define the variables and change the statement into an inequality.

A family has more dogs than cats.

Let $x$ be the number of dogs

Let $y$ be the number of cats

$x > y$

Ex: Define the variables and change the statement into an inequality.

A food truck sells hotdogs and hamburgers. Every day they sell at least 100 items.

Let $x$ be the number of hotdogs

Let $y$ be the number of hamburgers

$x + y \geq 100$

Optimization Unit – Step 2

Ex: Define the variables and change the statement into an inequality.

Reese collects international and domestic stamps. Reese has no more than twice as many domestic stamps as international stamps.

Let $x$ be the number of international stamps

Let $y$ be the number of domestic stamps

$y < 2x$

**Practice Questions**

3)Write an inequality for each of the following scenarios

a) Students are selling bracelets and necklaces as a fundraiser. They sell at least 5 more bracelets than necklaces.

b) Sidney is making homemade soaps to sell at a local market. Sidney makes citrus scented soap and lavender scented soap. Sidney expects to sell fewer than 100 total soaps.

c) Ms. Stinger is selling large and small jars of honey. She can make no more than twice as many small jars as large jars.

**Practice Problems**

4) A manufacturing plant produces cars and truck. Define the variables and translate each of the following statements into equalities or inequalities.

a) A maximum of 200 vehicles are produced day.

b) The plant must produce at least 100 vehicles each day.

c) Fewer than twice as many cars as trucks are produced.

d) The plant produces at least 40 cars each day.

e) The number of trucks produced must not exceed 150.

Optimization Unit – Step 3

**1.4 OPTIMIZATION STEP 3**

After turning statements into inequalities we need to graph those inequalities. Before we can do that, we must rearrange them into $y = ax + b$ form so we can find the initial value and the slope.

However, we are going to keep the inequality symbol ($<, >, \leq, or \geq$). We can rearrange inequalities just like we would rearrange equations with $=$, unless we multiply or divide by a negative number.

Ex: Define the variables, turn the statement into an equation, and rearrange into $y = ax + b$ form.
A bag contains red and blue marbles. There are at least 8 marbles in the bag.

Let $x$ be the number of red marbles
Let $y$ be the number of blue marbles

$x + y \geq 8$
$-x \quad\quad -x$

$y \geq -x + 8$

Ex: Define the variables, turn the statement into an inequality, and rearrange into $y = ax + b$ form.
A bag contains red and blue marbles. There are no more than twice as many red marbles as blue marbles.

Let $x$ be the number of red marbles
Let $y$ be the number of blue marbles

$x \leq 2y$

$2y \geq x$

$y \geq \frac{1}{2}x$

**Practice Problems**

1) Define the variables, turn the statement an inequality and, and rearrange into $y = ax + b$ form.

a) Students are holding a bake sale. They sell cookies and cupcakes. No more than 75 treats are sold.

b) John collects stuffed dinosaurs and stuffed bears. He has at least three times as many dinosaurs as bears.

c) John collects stuffed dinosaurs and stuffed bears. He has fewer than half as many bears as dinosaurs.

If an inequality only has one variable (either x or y, but not both), make sure the variable is on the left side of the inequality.

Ex: Define the variables, turn the statement into an inequality, and rearrange into $y = ax + b$ form.
In a school, students choose between art and music. At least 200 students sign up for music.

Let $x$ be the number of students in art

Let $y$ be the number of students in music

$y \geq 200$

**Practice Problems**

2) A café is selling tea and coffee. Define the variables, translate each of the following statements into inequalities, and rearrange into $y = ax + b$ form.

a) The café sells no more than 700 drinks in a day.

b) Every day the café sells at least 350 coffees.

c) The café sells fewer than twice as many coffees as teas.

d) A maximum of 200 teas are sold each day.

Optimization Unit – Step 4
**1.5 OPTIMIZATION STEP 4**

There are 3 steps to graphing an inequality (once it's been rearranged into $y = ax + b$ form).

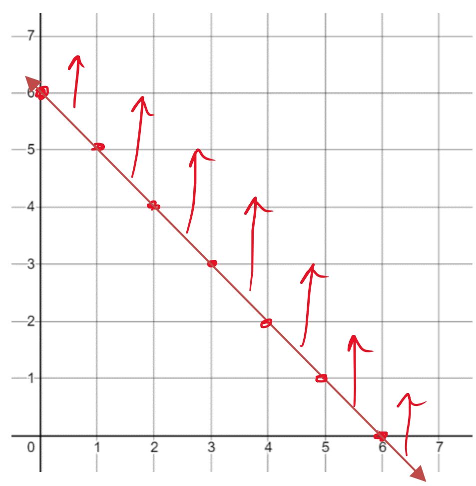
1) Determine whether to use a solid line or a dotted line.
    a. Use a solid line if $\leq$ or $\geq$
    b. Use a dotted line if $<$ or $>$
2) Graph as usual (putting a dot on the y-axis at b and then using slope to find a second point).
3) Shade either above the line or below the line
    a. Shade above the line (draw arrows straight up) if $> or \geq$
    b. Shade below the line (draw arrows straight down) if $< or \leq$

Ex: Define the variables, turn the statement into an equation, rearrange into $y = ax + b$ form and graph.
A bag contains red and blue marbles. There are at least 6 marbles in the bag.

Let $x$ be the number of red marbles
Let $y$ be the number of blue marbles.

$x + y \geq 6$

$y \geq -x + 6$



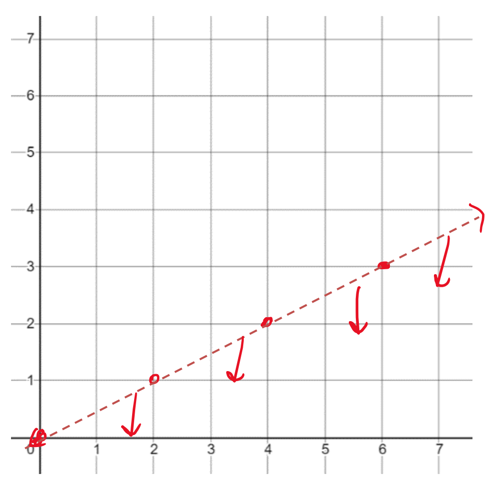

Ex: Define the variables, turn the statement into an equation, rearrange into $y = ax + b$ form and graph.
A bag contains red and blue marbles. There are more than twice as many red marbles as blue marbles.

Let $x$ be the number of red marbles.
Let $y$ be the number of blue marbles.

$x > 2y$

$2y < x$

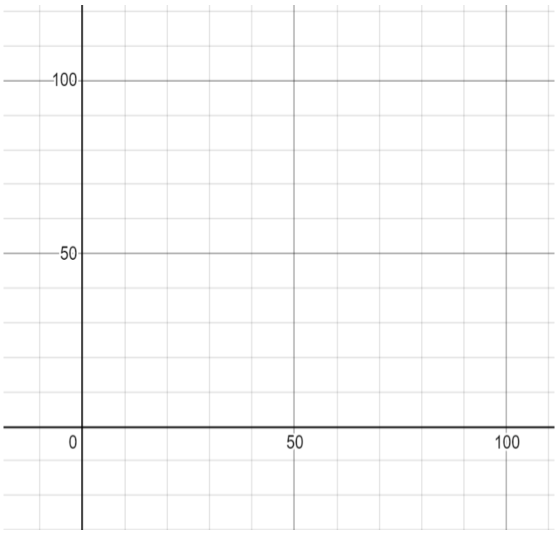
$y < \frac{1}{2}x$



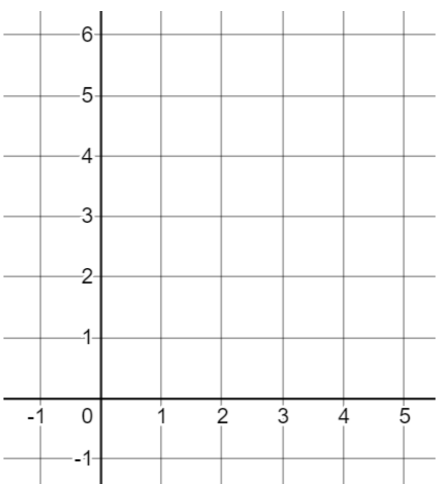
**Practice Problems**

1) Define the variables, turn the statement into an equation, rearrange into $y = ax + b$ form and graph.

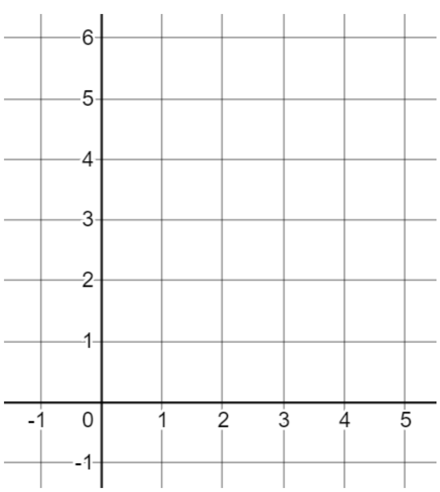
a) Students are holding a bake sale. They sell cookies and cupcakes. More than 75 treats are sold.




b) John collects stuffed dinosaurs and stuffed bears. He has at least three times as many dinosaurs as bears.




c) John collects stuffed dinosaurs and stuffed bears. He fewer than half as many bears as dinosaurs.



Optimization Unit – Step 4

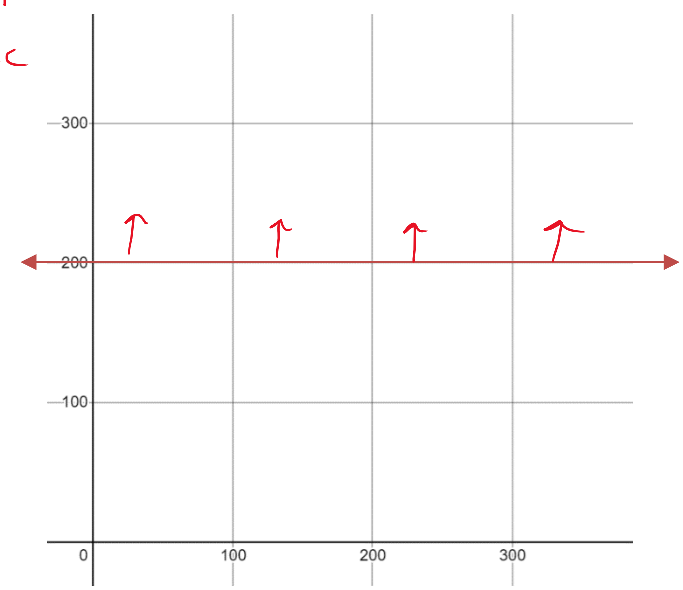
If you only have one variable, isolate the variable on the left side and graph and shade as usual, except:

- If $x >$ or $x \geq$ shade to the right of the line (or draw arrows to the right)
- If $x <$ or $x \leq$ shade to the left of the line (or draw arrows to the left)

Ex: Define the variables, turn the statement into an inequality, and rearrange into $y = ax + b$ form.
In a school, students choose between art and music. At least 200 students sign up for music.

Let $x$ be the number of students in art
Let $y$ be the number of students in music

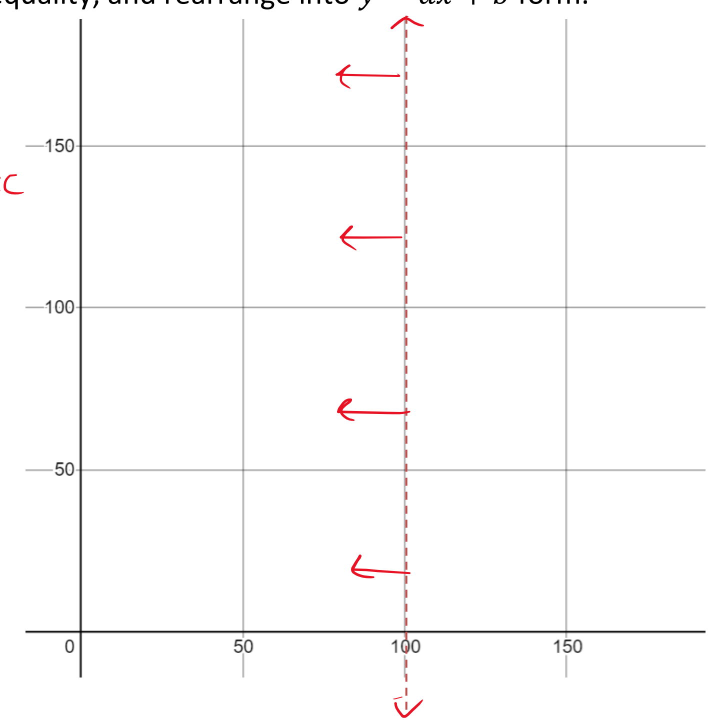
$y \geq 200$




Ex: Define the variables, turn the statement into an inequality, and rearrange into $y = ax + b$ form.
In a school, students choose between art and music.
Fewer than 100 students sign up for art.

Let $x$ be the number of students in art
Let $y$ be the number of students in music

$x < 100$

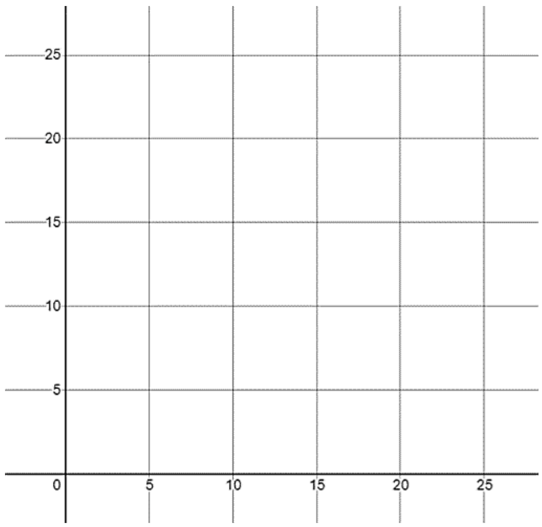


**Practice Problems**

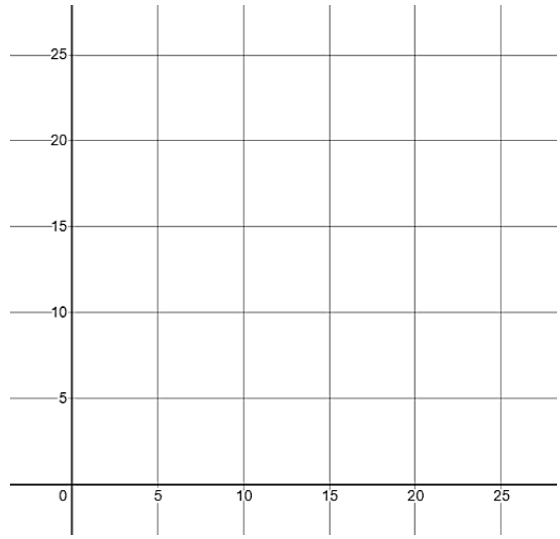
2) Define the variables, translate each of the following statements into inequalities, and rearrange into $y = ax + b$ form and graph for the scenario below.

Lisa, a Grade 11 student, is fundraising for Prom. She sells strawberry baskets and flower baskets.

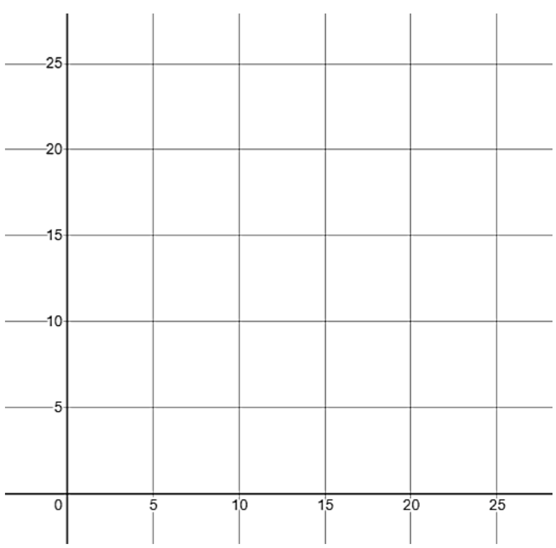
a) A maximum of 24 baskets can be sold.



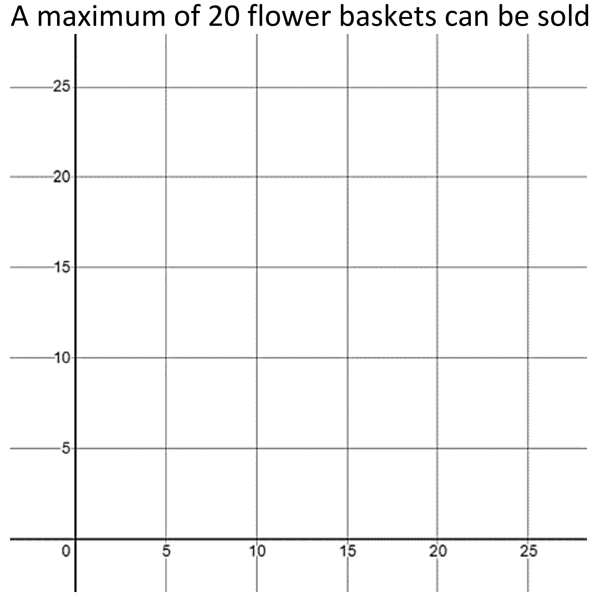

b) A minimum of 5 strawberry baskets must be sold.




c) The number of flower baskets must be at least triple the number of strawberry baskets sold.




d) A maximum of 20 flower baskets can be sold.



## 1.6 OPTIMIZATION STEP 5

When we have more than one linear inequality, we can graph them all together. This is a system of linear inequalities.

The solution to a system of inequalities is where all the shading overlaps.

A **polygon of constraints** is when the overlapping shading is bound on all sides by a line.

There are 3 steps to graphing a system of inequalities:

4) Graph and shade all lines (instead of shading, it can help to use arrows).
5) Determine the polygon of constraints by determining the area where all shading overlaps.
6) Shade the polygon of constraints.

Ex: Determine the polygon of constraints given the following scenario:
A bag contains red and blue marbles.

- There are no more than 4 blue marbles.
- There is at least 1 red marble.
- There is a maximum of 6 marbles.

Let x be the number of red marbles
Let y be the number of blue marbles

① $y \leq 4$

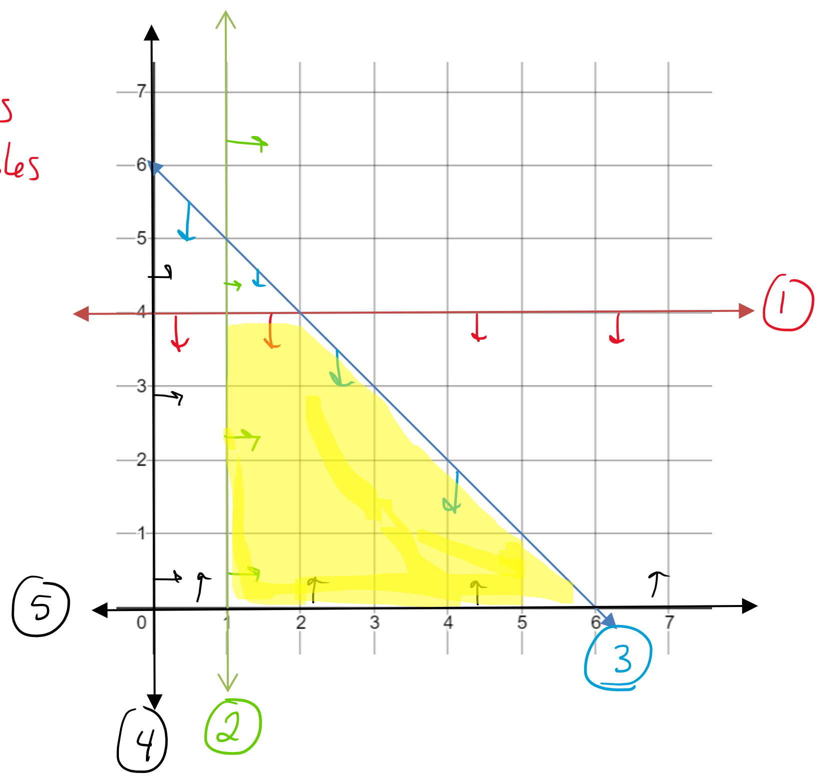
② $x \geq 1$

③ $x + y \leq 6$
$\quad -x \qquad -x$

$y \leq -x + 6$

④ $x \geq 0$

⑤ $y \geq 0$

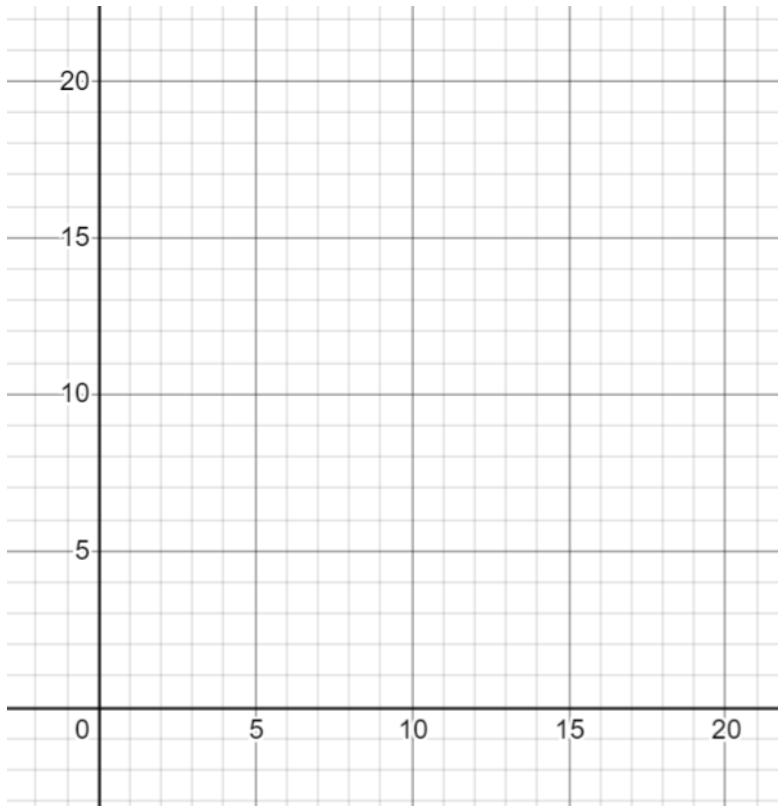


**Practice Problems**

1) Determine the polygon of constraints given the following scenario.

John loves his stuffed animals. His collection has stuffed bears and stuffed dinosaurs.

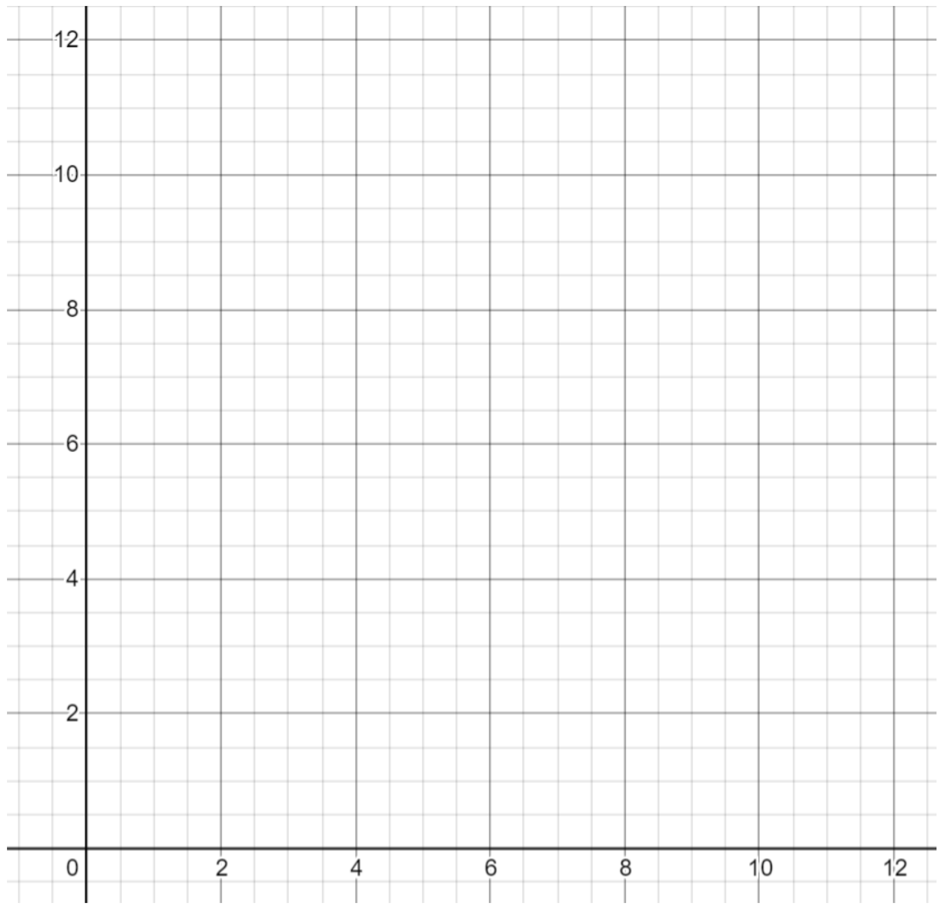
- John has a minimum of 10 stuffed animals.
- John has a maximum of 20 stuffed animals.
- John has less than or equal to 6 bears.
- John has at least 5 more dinosaurs than bears.



2) Determine the polygon of constraints given the following scenario.

Sandra and Jane run a pet spa. They offer two services: dog grooming and cat grooming.

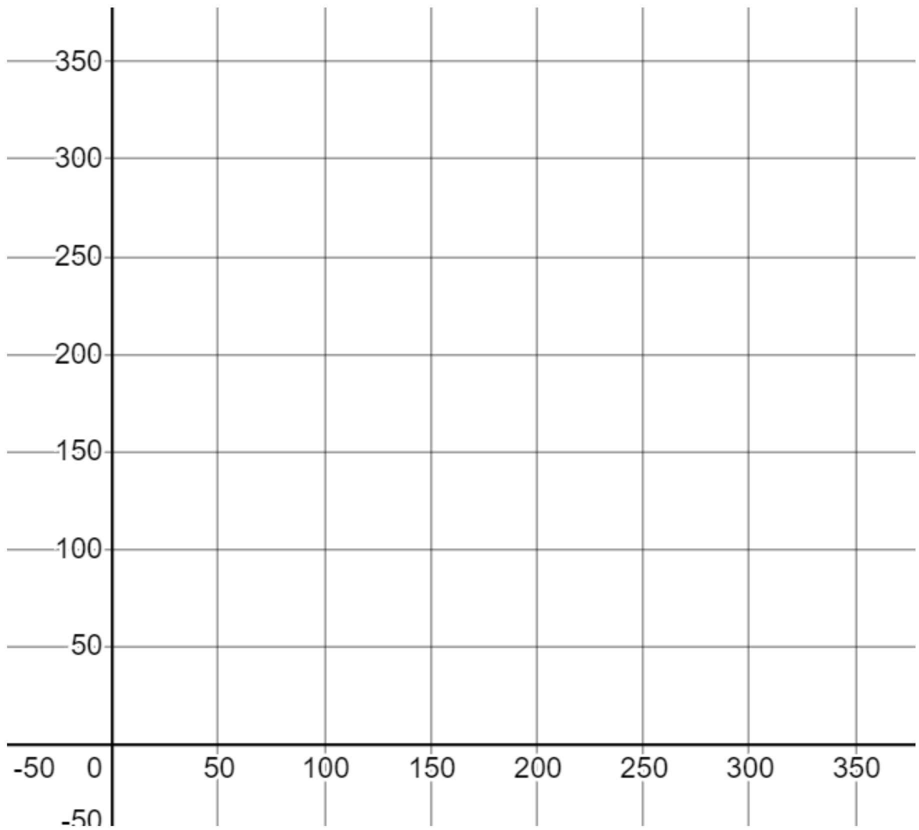
- They will bathe a maximum of 12 cats per day.
- They can bathe no more than 8 dogs per day.
- They will bathe a maximum of 3 times as many cats as dogs.



3) Determine the polygon of constraints given the following scenario.

To raise funds, members of an association organize a concert. They want to reserve some seats for donors and the other seats will be for general admission.

- The theater contains a maximum of 300 seats.
- There must be at least 3 times as many seats for general admission as there are seats reserved for donors.
- There must be at least 50 seats for donors.
- There is a maximum of 250 seats for general admission.



**1.7 OPTIMIZATION STEP 6**

Now that we have the polygon of constraints, we need to find the vertices. That is, we need to find the (x, y) of each vertex (corner) of the polygon of constraints.

There are 3 steps to finding the vertices:

1) Label each vertex of the polygon of constraints (we usually use A, B, C, etc.).
2) Identify the two lines that go through each vertex and change inequalities to =
3) Solve the system

| If the lines that intersect to form the vertex are: | Then use: |
|---|---|
| 2 equations each including both variables | Comparison |
| 1 equation with both variables and 1 equation with one variable | Substitution |
| 2 equations each including one variable | No calculations necessary |

Ex: Determine the vertices of the polygon of constraints given the following scenario:

A bag contains red and blue marbles.

- The bag contains a maximum of 6 marbles. ① $x+y \leq 6 \Rightarrow y \leq -x+6$
- The bag contains a minimum of 2 marbles. ② $x+y \geq 2 \Rightarrow y \geq -x+2$
- There are at least as many red marbles as blue marbles. ③ $x \geq y \Rightarrow y \leq x$
- The bag contains at most 5 red marbles. ④ $x \leq 5$

⑤ $x \geq 0$

⑥ $y \geq 0$

Let $x$ be the number of red marbles

Let $y$ be the number of blue marbles

**A**

$y = -x+6$

$y = x$

$x = -x+6$

$2x = 6$

$x = 3$

$y = 3$

$(3,3)$

**B**

$x = 5$

$y = -x+6$

$y = -5+6$

$y = 1$

$(5,1)$

**C**

$y = 0$

$x = 5$

$(5,0)$

**D**

$y = 0$

$y = -x+2$

$0 = -x+2$

$x = 2$

$(2,0)$

**E**

$y = -x+2$

$y = x$

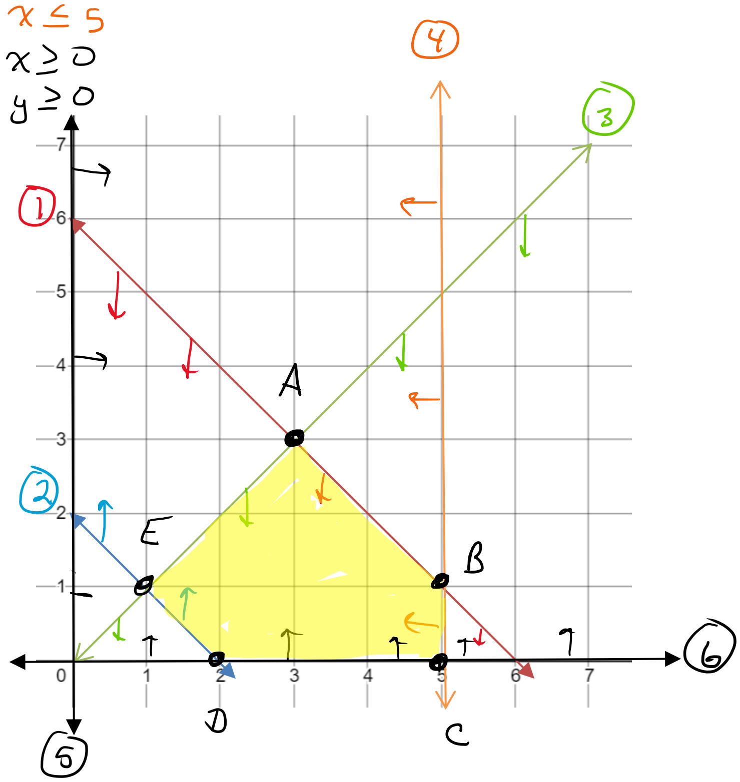
$x = -x+2$

$2x = 2$

$x = 1$

$y = 1$

$(1,1)$

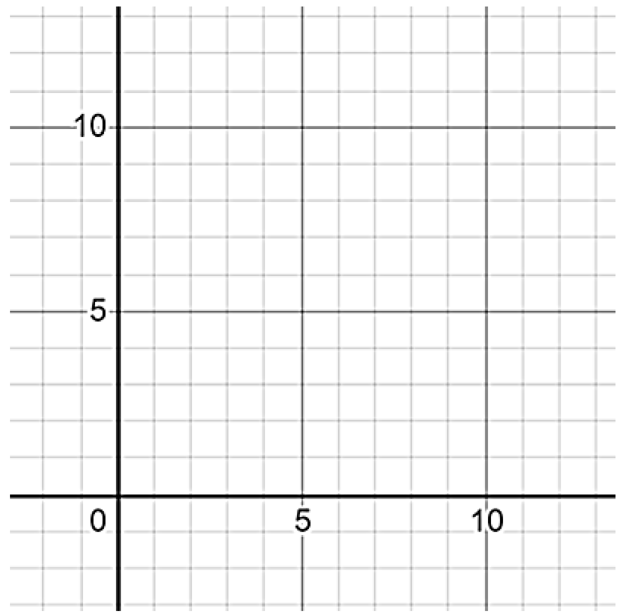


**Practice Question**

1) Determine the vertices of the polygon of constraints given the following scenario:

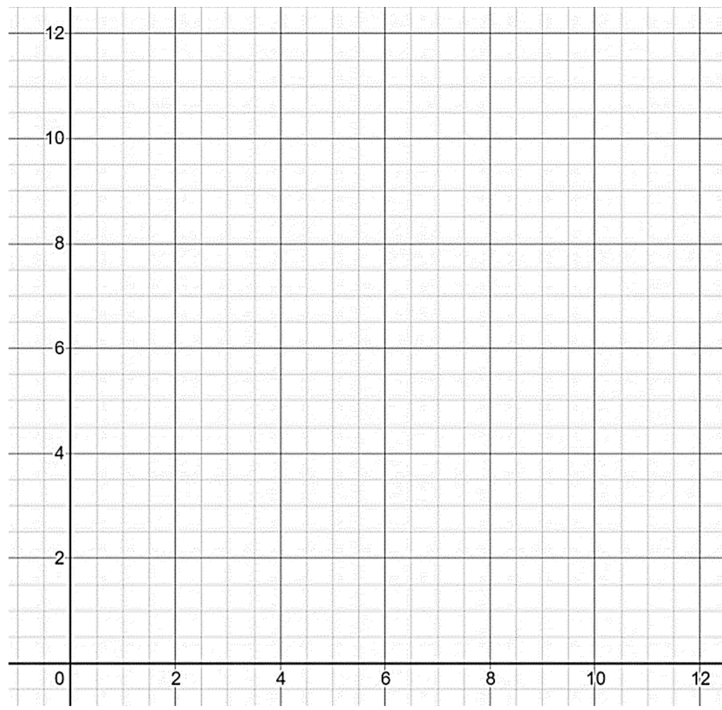
John loves his stuffed animals. His collection has stuffed bears and stuffed dinosaurs.

- John has a maximum of 10 stuffed animals.
- John has less than or equal to 4 bears.
- John has at least 1 more dinosaur than bear.



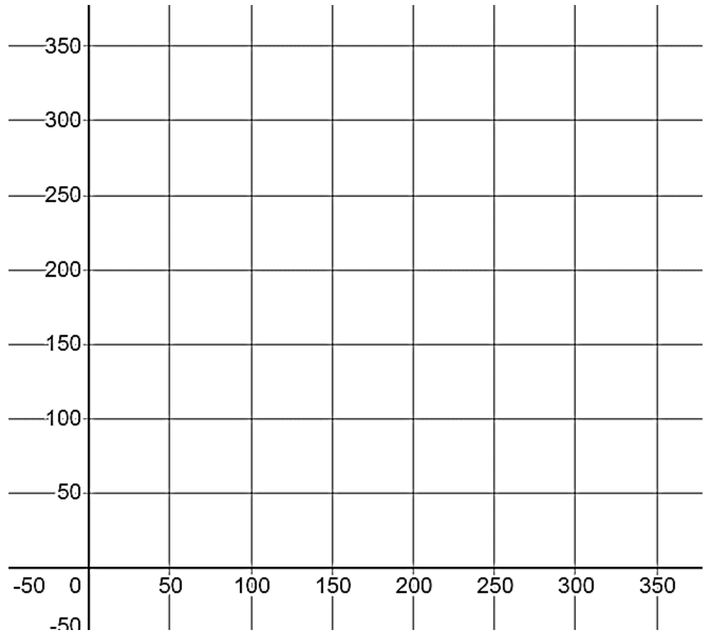
2) Sandra and Jane run a pet spa. They offer two services: dog grooming and cat grooming.

- They will bathe a maximum of 12 cats per day.
- They can bathe no more than 8 dogs per day.
- They will bathe a maximum of 3 times as many cats as dogs.



3)To raise funds, members of an association organize a concert. They want to reserve some seats for donors and the other seats will be for general admission.

- The theater contains a maximum of 300 seats.
- There must be at least 3 times as many seats for general admission as there are seats reserved for donors.
- There must be at least 50 seats for donors.
- There is a maximum of 250 seats for general admission.



Optimization Unit – Steps 7-10

**1.8 STEPS 7-10**

The **target objective** is the search for the optimal solution. It is either the search for the highest value (maximum) or lowest value (minimum). The optimal value is obtained by using the **optimizing function**.

In each optimization question, you will be given a statement about money (cost, profit, etc.). This will become the optimizing function.

When you substitute each vertex into the optimizing function (one at a time), one vertex will give you the maximum and one vertex will give you the minimum.

- **Step 7** is to identify the target objective and write the optimizing function. Go back to the question and pick out the sentence that has to do with money. Turn this into an equation.
  **Be Careful! You won't have to use a statement about money until this step, so ignore it when you are creating your graph and finding your vertices.*

- **Step 8** is to use the optimizing function at each vertex. Take each vertex, one at a time, and replace the x and y in the optimizing function with the (x, y) coordinates of the vertex.

- **Step 9** is to choose the maximum or minimum depending on the vertex. One of the vertices will give you the maximum. One of the vertices will give you the minimum.

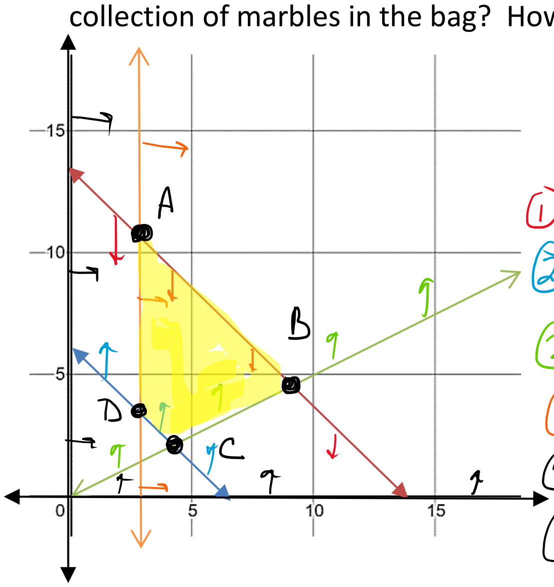
- **Step 10** is to write a concluding statement answering the question. The question may ask of the maximum, minimum, or the number of items that give you a maximum or minimum.

Ex: Determine the vertices of the polygon of constraints given the following scenario:

A bag contains red and blue marbles.

1 • The bag contains a maximum of 12 marbles.
2 • The bag contains a minimum of 6 marbles.
3 • There are no more than twice as many red marbles as blue marbles.
4 • The bag contains at least 3 red marbles.

If each red marble is worth \$1.50 and each blue marble is worth \$3.00, what is the maximum value of the collection of marbles in the bag? How many of each marble do you need to have the maximum value?




Let $x$ be the number of red marbles

Let $y$ be the number of blue marbles

(1) $x+y \leq 12 \Rightarrow y \leq -x+12$

(2) $x+y \geq 6 \Rightarrow y \geq -x+6$

(3) $x \leq 2y \Rightarrow 2y \geq x \Rightarrow y \geq \frac{1}{2}x$

(4) $x \geq 3$

(5) $x \geq 0$

(6) $y \geq 0$

Optimization Unit – Steps 7-10

A

$x = 3$
$y = -x + 12$
$y = -3 + 12$
$y = 9$
$(3, 9)$

B

$y = -x + 12$
$y = \frac{1}{2}x$
$\frac{1}{2}x = -x + 12$
$1.5x = 12$
$x = 8$
$y = 0.5(8)$
$y = 4$
$(8, 4)$

C

$y = \frac{1}{2}x$
$y = -x + 6$
$-x + 6 = \frac{1}{2}x$
$6 = 1.5x$
$4 = x$
$y = -x + 6$
$y = -4 + 6$
$y = 2$
$(4, 2)$

D

$x = 3$
$y = -x + 6$
$y = -3 + 6$
$y = 3$
$(3, 3)$

T.O. Maximize value
O.F. Value $= 1.5x + 3y$

A $(3, 9)$

$V = 1.5x + 3y$
$V = 1.5(3) + 3(9)$
$V = 4.5 + 27$
$V = 31.5$
$V = \$31.50$

B $(8, 4)$

$V = 1.5x + 3y$
$V = 1.5(8) + 3(4)$
$V = 12 + 12$
$V = 24$
$V = \$24.00$

C $(4, 2)$

$V = 1.5x + 3y$
$V = 1.5(4) + 3(2)$
$V = 6 + 6$
$V = 12$
$V = \$12.00$

D $(3, 3)$

$V = 1.5x + 3y$
$V = 1.5(3) + 3(3)$
$V = 4.5 + 9$
$V = 13.5$
$V = \$13.50$

This is the biggest

The maximum value of the collection is \$31.50, and you need 3 red and 9 blue marbles.

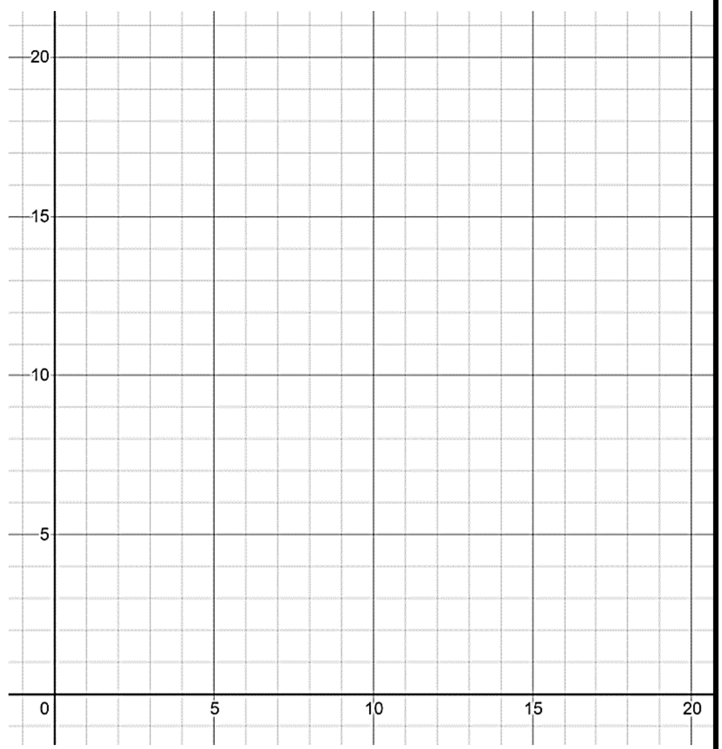
**Practice Questions**

1) Determine the vertices of the polygon of constraints, the optimizing function, and the optimal solution.

John loves his stuffed animals. His collection has stuffed bears and stuffed dinosaurs.

- John has a maximum of 20 stuffed animals.
- John knows he has at least 10 stuffed animals.
- John has less than or equal to 6 bears.
- John has at least 5 more dinosaurs than bears.

Given that each bear is worth $10 and each dinosaur is worth $5, what is the minimum value of John's collection and how many of each animal would John have if his collection was worth the minimum value?

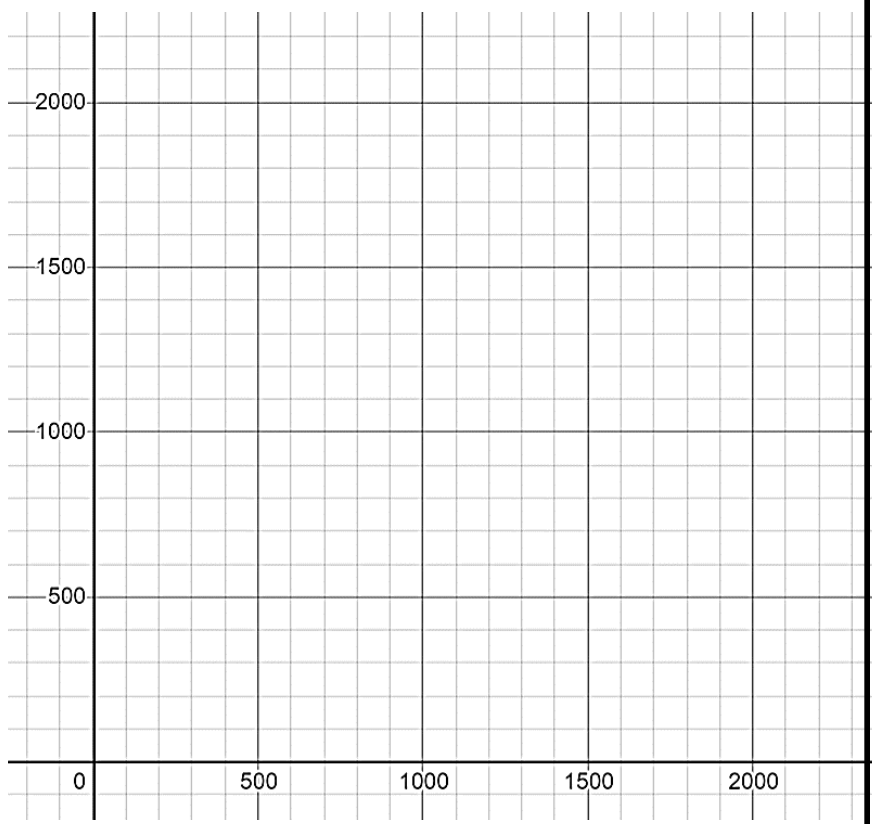


2) Determine the vertices of the polygon of constraints, the optimizing function, and the optimal solution.

A car manufacturer builds compact cars and minivans and wants to maximize its weekly profit. The profit generated from each car is $4000 and the profit generated from each minivan is $10 000.

- The manufacturer's weekly production capacity is 2100 at most.
- The manufacturer must build at least 1000 compact cars weekly.
- The manufacturer must build at least 200 minivans weekly.
- The number of compact cars built each week must be at least twice as many as the number of minivans built each week.

What is the maximum profit the manufacturer can earn weekly?



3) Determine the vertices of the polygon of constraints, the optimizing function, and the optimal solution.

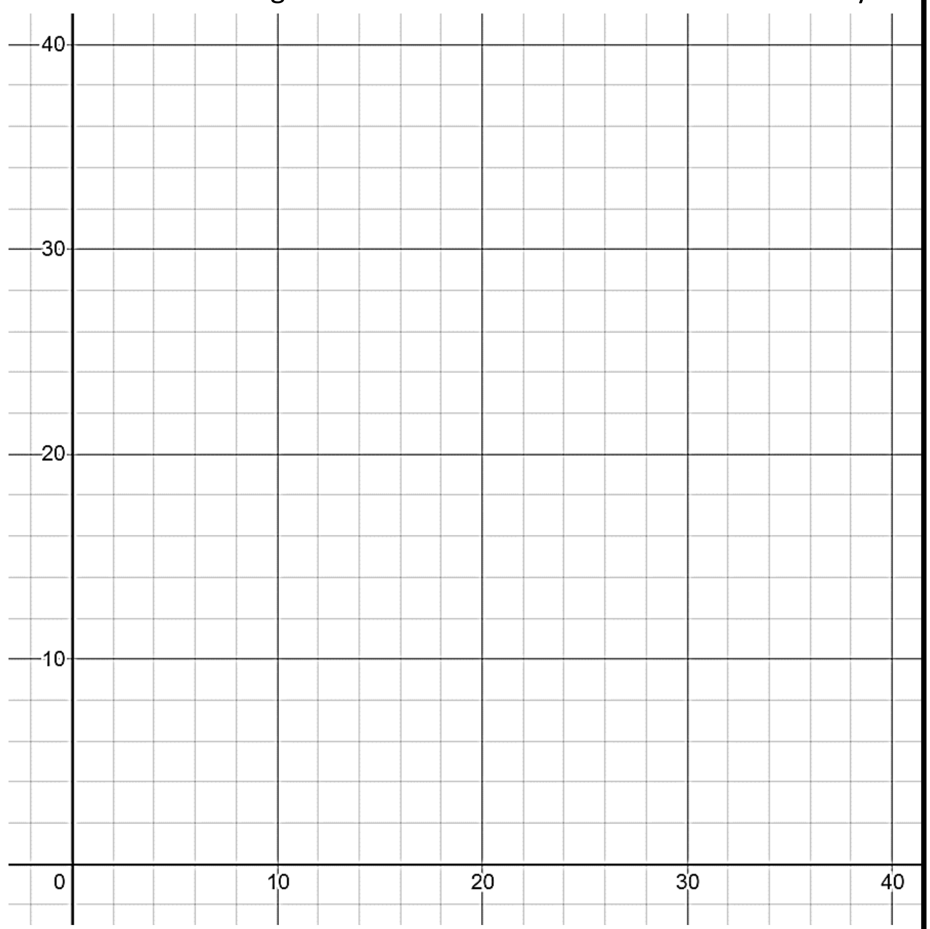
In order to treat a patient, a doctor decides to administer a treatment combining two medications, A and B. The side effects associated with these medications force the doctor to respect the following constraints:

- The dosage of medication A must be at least 5 mg.
- The dosage of medication A cannot exceed 15 mg.
- The dosage of medication B must be at least 8 mg.
- The dosage of medication B cannot exceed 25 mg.
- The total dosage of medication cannot exceed 35 mg.

The doctor wants the medications to be as effective as possible. Efficacy can be determined by using the following optimizing function: $Efficacy = 0.0305x + 0.025y$ where:

- x is the amount of medication A
- y is the amount of medication B

What combination of medications should the doctor give in order to achieve the maximum efficacy?



**1.9 OPTIMIZATION COMPLICATIONS: DECIMALS, DOTTED LINES, and TIES**

There are three potential complications to optimization: decimals, dotted lines, and ties.

**Decimals**

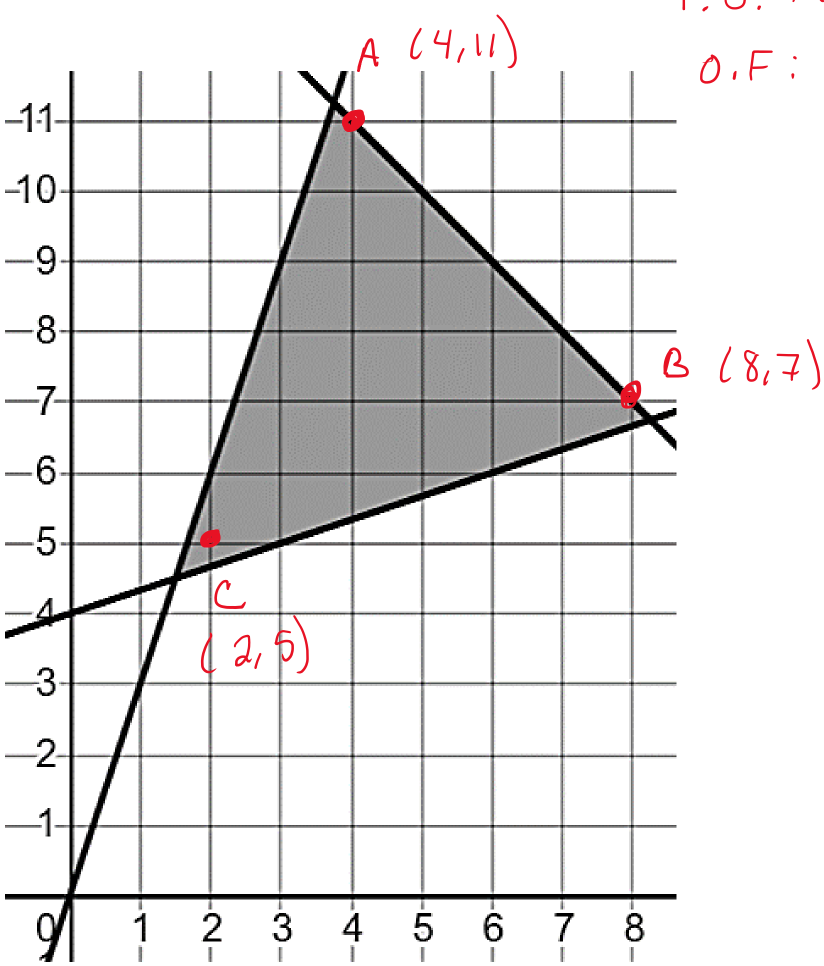
In optimization, the vertices always need to be whole numbers (not decimals). This is because we are talking about things that can only be produced or sold (for example) in whole units.

When we solve for the vertices, we always need to find a whole number (not a decimal). Sometimes we will get a decimal. When this happens, find a point near the vertex where x and y are both whole numbers either in the shaded area or on a solid line. Use that point instead of the decimals.

A school is holding a car wash and charges $5 to wash a car (x) and $8 to wash a truck (y). The students can only wash a limited number of vehicles per hour. The polygon of constraints is shown in the graph below. What is the maximum value the students could earn per hour?

T.O. Maximize Value

O.F: Value = $5x + 8y$




A (4,11)

$V = 5(4) + 8(11)$

$V = 20 + 88$

$V = 108$

$V = \$108.00$

This is the biggest

B (8,7)

$V = 5(8) + 8(7)$

$V = 40 + 56$

$V = 96$

$V = \$96.00$

C (2,5)

$V = 5(2) + 8(5)$

$V = 10 + 40$

$V = 50$

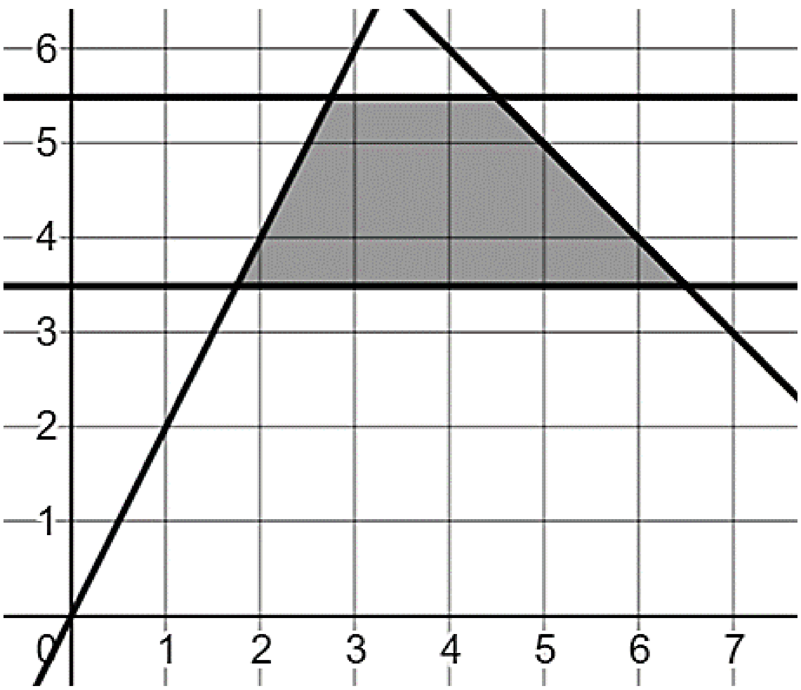
$V = \$50$

∴ the maximum they could earn is $108.00

**Practice Question**

1) Reese wants to maximize the value of his marble collection. Reese collects red and blue marbles, but he has limited space and money to purchase marbles. The possible combinations of red marbles (x) and blue marbles (y) are shown in the polygon of constraints below.

Given that red marbles are worth $2 and blue marbles are worth $1.50, what is the maximum value of Reese's collection?



Optimization Unit – Complications: Decimals, Dotted Lines, and Ties

**Dotted Lines**

Dotted lines (which means the inequality has a $>$ or $<$ instead of a $\geq$ or $\leq$) presents another complication. If a vertex is on a dotted line, it cannot be the solution. This means the vertex itself is not included in the solution set, so cannot be the answer. However, a point close to the vertex could be the answer.

We will first solve this as if the line was not dotted. If the optimal solution is on a vertex with a dotted line, then we find a point close to that vertex and check to see if it's also an optimal solution. If not, we select a different vertex.

Ex: A coffee shop sells tea and coffee. The number of teas (x) and coffees (y) sold each hour is given by the polygon of constraints below.

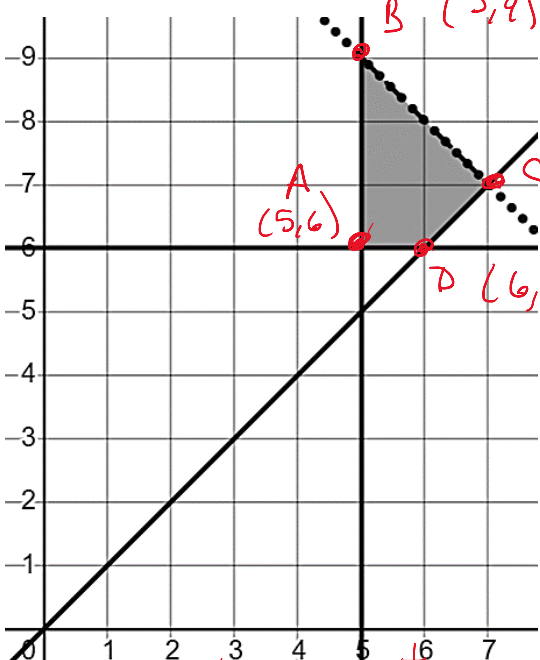
If the coffee shop earns a profit of \$0.75 for each tea sold and \$1.00 for each coffee sold. How many teas and coffees must the shop sell each hour in order to earn the maximum profit?

B* (5,9)

A (5,6)

C* (7,7)

D (6,6)




T.O. Maximize profit

O.F Profit = 0.75x + 1y

A (5,6)

P = 0.75(5) + 1(6)

P = 3.75 + 6

P = 9.75

B* (5,9)

P = 0.75(5) + 1(9)

P = 3.75 + 9

P = 12.75

Biggest!

C* (7,7)

P = 0.75(7) + 1(7)

P = 5.25 + 7

P = 12.25

D (6,6)

P = 0.75(6) + 1(6)

P = 4.5 + 6

P = 10.5

B is the max, but cannot be the answer (dotted line)

Try (5,8)

P = 0.75(5) + 1(8)

P = 3.75 + 8

P = 11.75

NO LONGER THE BIGGEST.

C is has biggest

Now C is the max, but cannot be the answer (dotted line)

try (6,7)

P = 0.75(6) + 1(7)

P = 4.5 + 7

P = 11.5

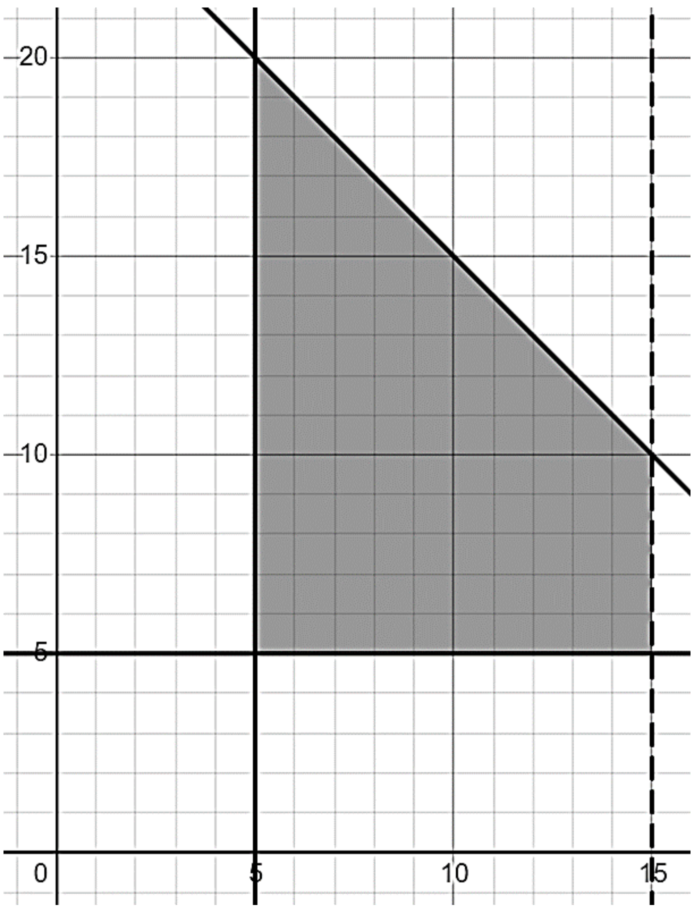
THE BIGGEST!

∴ Max profit is \$11.50 per hour.

**Practice Questions**

2) A pizza shop sells whole pizzas (x) and slices of pizza (y). The possible combinations of whole pizzas and slices of pizza sold per hour is given in the polygon of constraints below.

Given that the shop earns a profit of $4 per whole pizza and $1.50 per slice of pizza, how many slices should the store per hour in order to earn the maximum profit?



Optimization Unit – Complications: Decimals, Dotted Lines, and Ties

**Ties**

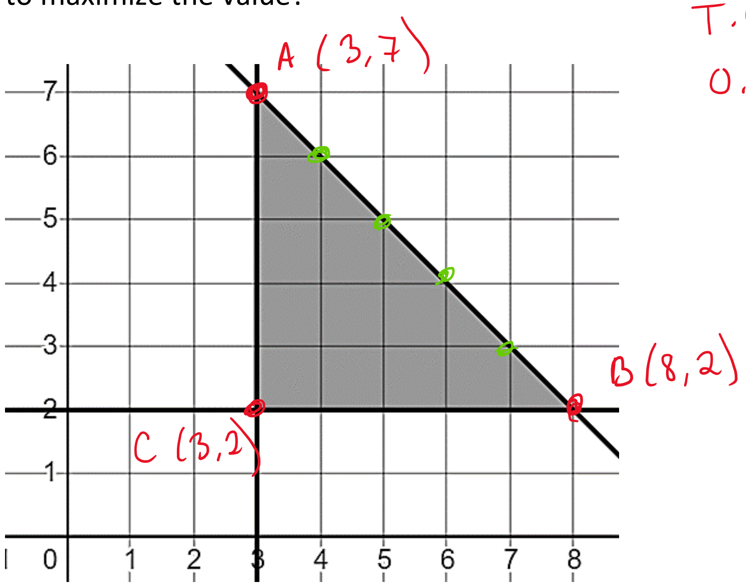
The final complication present in some optimization questions is when two vertices tie for a maximum or minimum after using the optimizing function.

When two vertices tie for the optimal value, they are both solutions to the scenario. In addition, all the points on the line connecting the two tied vertices are also optimal solutions.

Ex: Students at Philemon Wright are selling scented candles as a fundraiser for their annual trip. They are selling lavender scented candles (x) and vanilla scented candles (y). The possible combinations of lavender and vanilla candles sold each day are presented in the polygon of constraints below.

The profit for the lavender scented candles is \$3 and the profit for the vanilla scented candles is also \$3.

What is the maximum profit the students make each day and what are all the possible combinations of scents to maximize the value?




T.O. Maximize profit

O.F Profit = $3x + 3y$

A (3,7)
$P = 3(3) + 3(7)$
$P = 9 + 21$
$P = 30$

B (8,2)
$P = 3(8) + 3(2)$
$P = 24 + 6$
$P = 30$

TIE

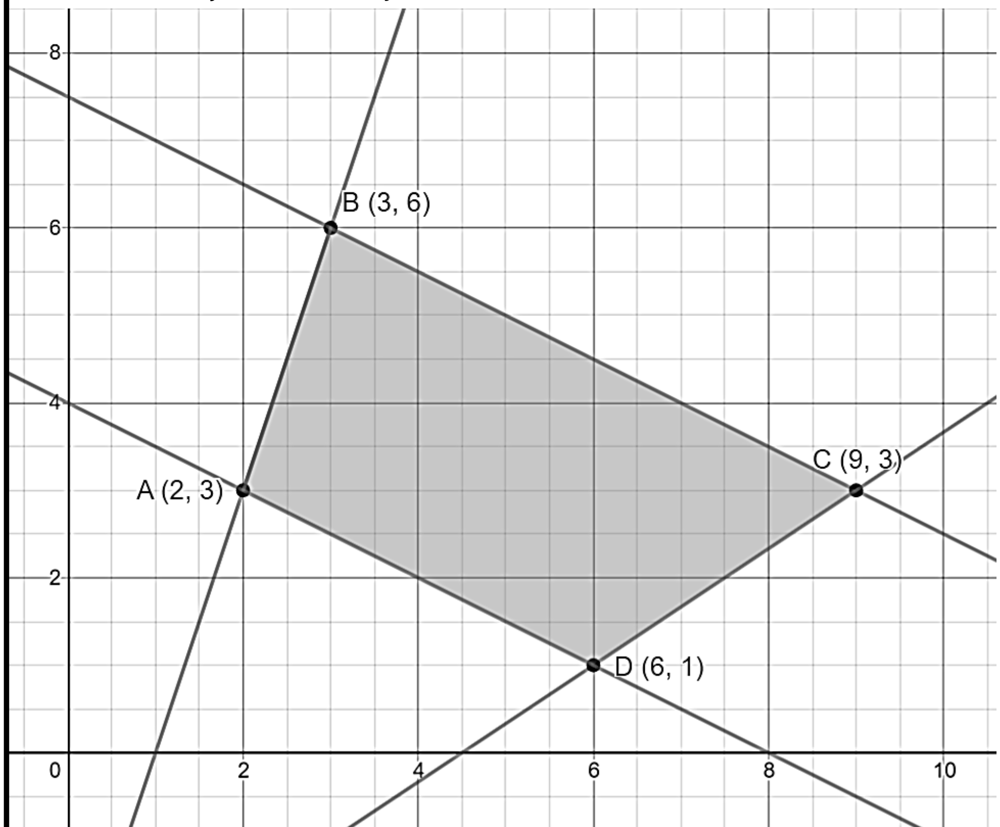
C
$P = 3(3) + 3(2)$
$P = 9 + 6$
$P = 15$

Max profit is 30

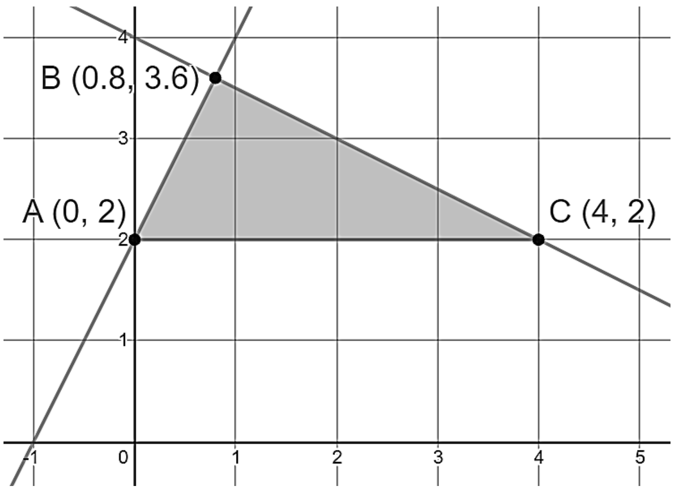
Max profit is \$30, and the possible combinations to get the max profit are: (3,7), (4,6), (5,5), (6,4), (7,3), and (8,2)

**Practice Questions**

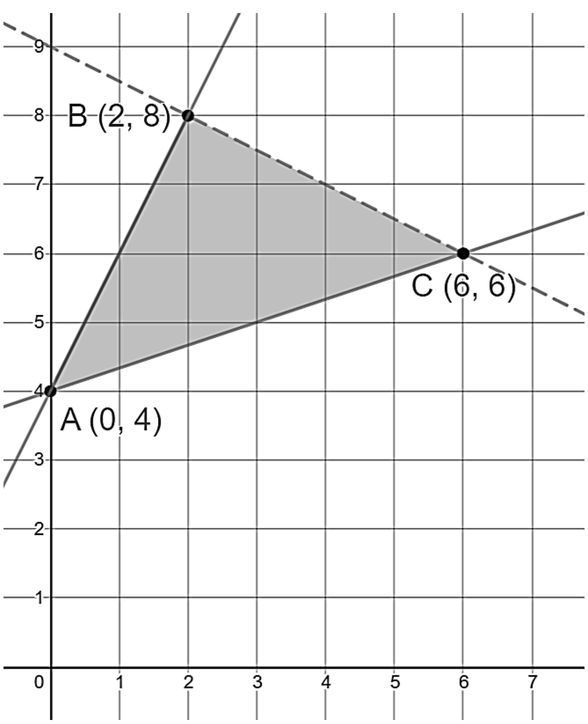
3) The polygon of constraints of an optimization question is represented on the graph below. The optimizing function is: $Profit = 3x + 6y$. How many points maximize this situation? What are they?



4) The polygon of constraints of an optimization question is represented on the graph below. The optimizing function is: $Cost = \$4x + \$2y$. What is the minimum cost in this scenario?

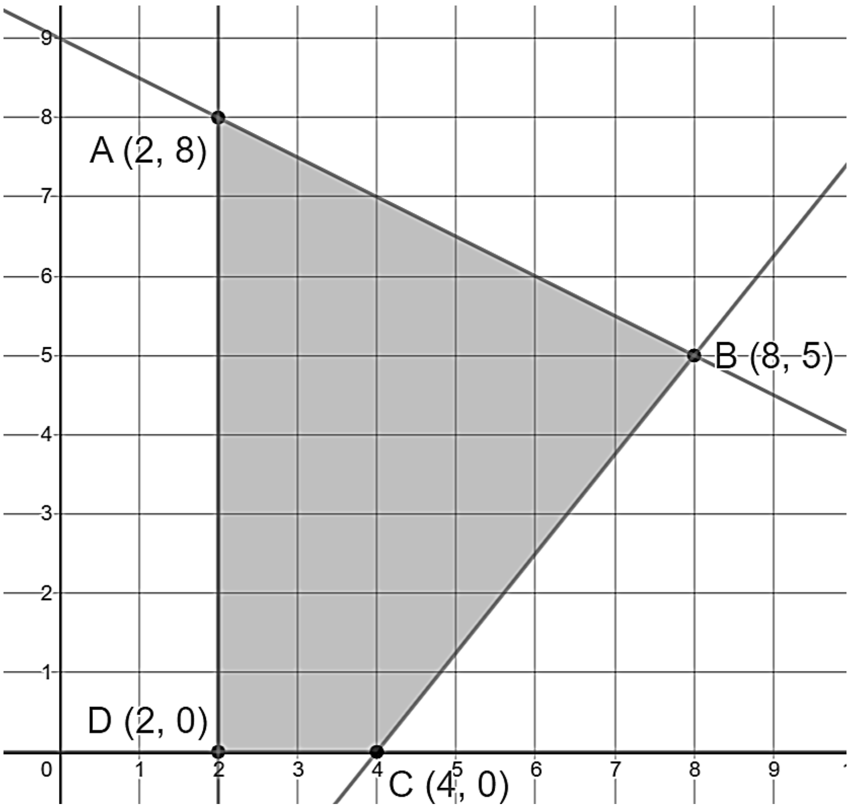



5) The polygon of constraints of an optimization question is represented on the graph below. The optimizing function is $Value = \$2x + \$10y$. What is the maximum value in this scenario?



6) The polygon of constraints of an optimization question is represented on the graph below. The optimizing function is $Value = 4x + 8y$.

What is the maximum value and how many points maximize the scenario?



Optimization Unit – Exam Style Questions
## 1.10 OPTIMIZATION EXAM STYLE QUESTIONS

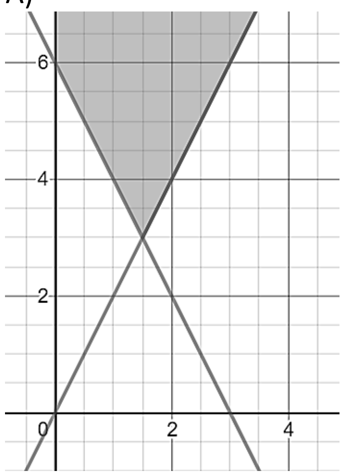
**Multiple Choice**

1) The system of inequalities below represents the constraints associated with an optimization situation.

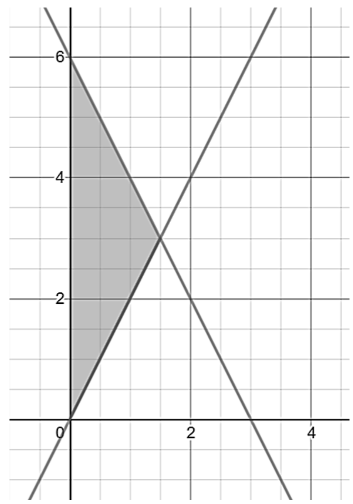
$$y \geq 0$$
$$x \geq 0$$
$$y \geq 2x$$
$$4x + 2y \leq 12$$

**Which of the following represents the solutions for this system of inequalities?**

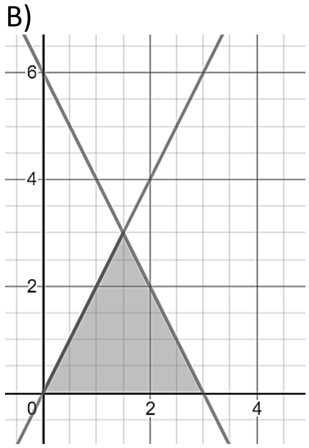
A)



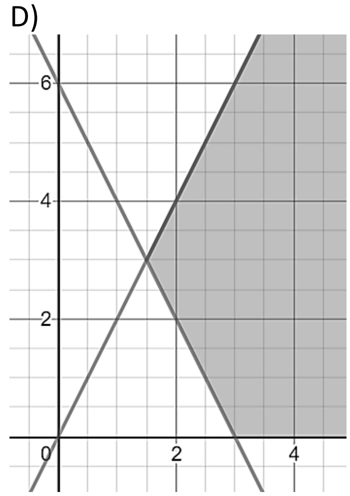

C



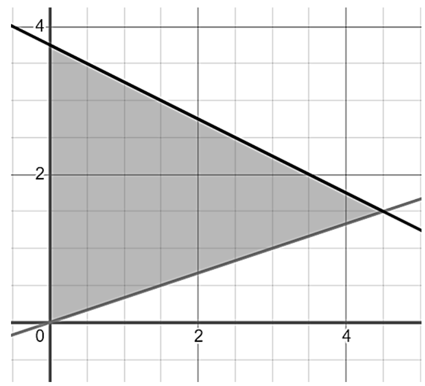

B)




D)



2) The graph below represents the polygon of constraints associated with an optimization situation.




**Which of the following system of inequalities corresponds with this optimization system?**

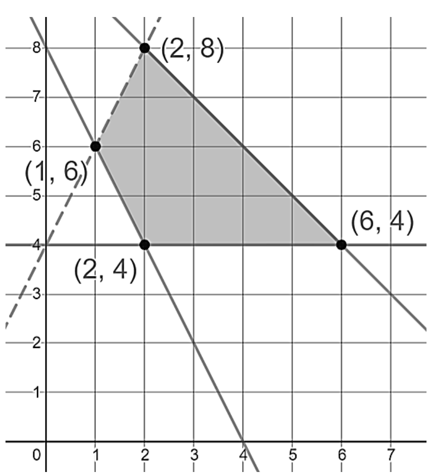
A)
$y \geq 0$
$x \geq 0$
$x \geq 3y$
$2x + 4y \leq 15$

B)
$y \geq 0$
$x \geq 0$
$x \leq 3y$
$2x + 4y \leq 15$

C)
$y \geq 0$
$x \geq 0$
$x \leq 3y$
$2x + 4y \geq 15$

D)
$y \geq 0$
$x \geq 0$
$x \geq 3y$
$2x + 4y \geq 15$

3) The polygon of constraints below is associated with an optimization situation.
The optimizing function is $Z = 100x + 100y$




How many points on the graph maximize the situation?

A) 1

B) 2

C) 4

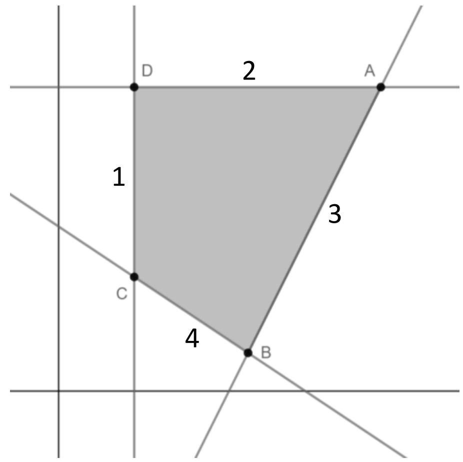
D) 5

Optimization Unit – Exam Style Questions

**Short Answer**

4) The constraints associated with an optimization situation are represented by the systems of inequalities and the polygon of constraints shown below. Each side of the polygon and its corresponding inequality are identified by the same number.

1) $x \geq 2$
2) $y \leq 8$
3) $y \geq 2x - 9$
4) $2x + 3y \geq 13$




**What are the coordinates of vertex B of this polygon of constraints?**

5) A high school is selling t-shirts and hoodies as a fundraiser for the class trip.
x: the number of t-shirts sold
y: the number of hoodies sold

**Translate the following statements into inequalities.**

- **The school will sell a maximum of 200 items.**
- **There will be at least twice as many t-shirts as hoodies sold.**

6) Each year Grade 11 students hold a car wash as a fundraiser for Prom. The revenue raised from the car wash is represented by the optimizing function $Z = 8x + 10y$

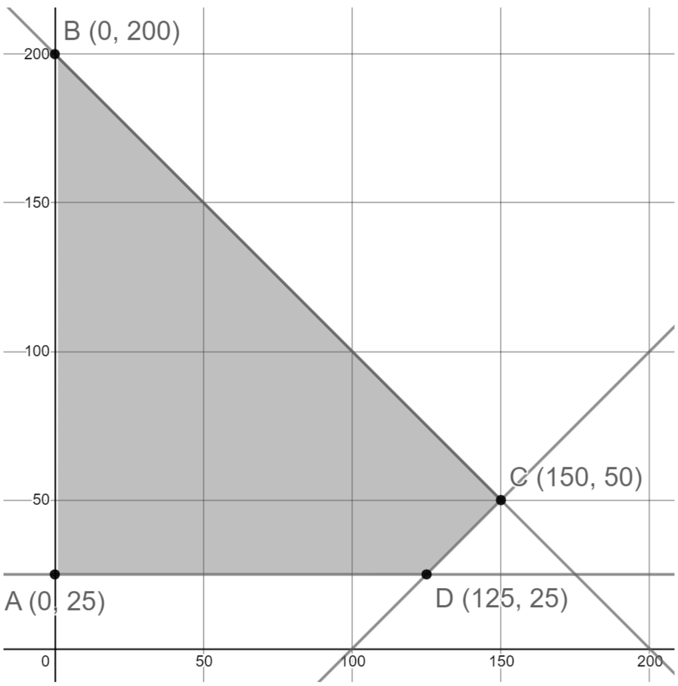
where: x is the number of cars washed

y is the number of trucks washed

The polygon of constraints below represents the combination of cars and trucks washed in a typical year.

This year, there is heavy construction near the car wash site. The organizers expected there will be fewer cars and trucks to wash as a result. The decrease in vehicles is represented by the inequality below.

$$x + y \leq 150$$



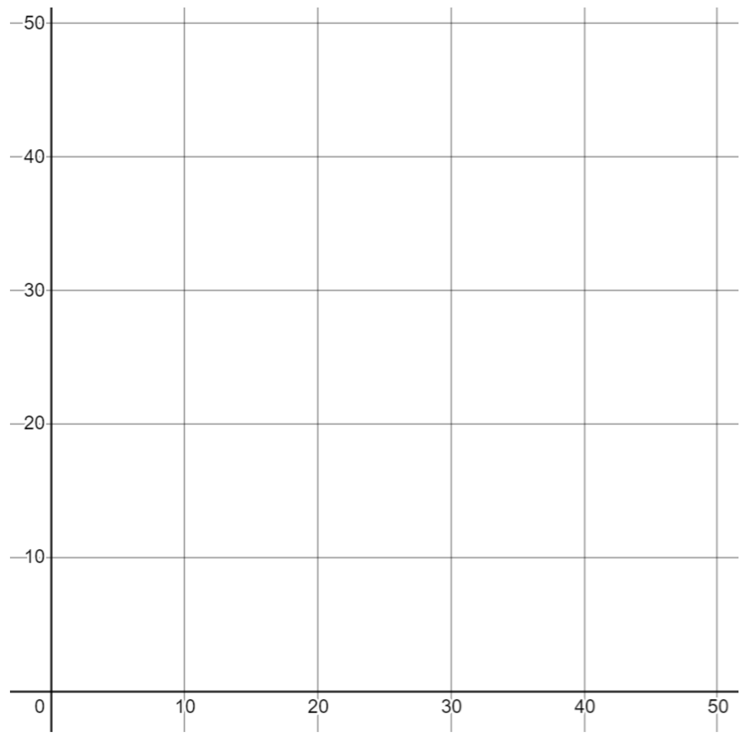

| Vertex | Revenue $Z = 8x + 10y$ |
|---|---|
| A (0, 25) | \$250 |
| B (0, 200) | \$2000 |
| C (150, 50) | \$1700 |
| D (125, 25) | \$1250 |

**By how much will the students' expected maximum revenue at the car wash decrease as a result of the construction?**

**Long Answer**

7) Mackenzie is selling tulip bulbs as part of the school band fundraiser. They will offer a choice between standard red tulip bulbs and special orange tulip bulbs. Mackenzie's sales are limited by the following constraints:
   - Mackenzie can sell a maximum of 40 bulbs per day.
   - Mackenzie must sell a maximum of 10 red bulbs per day.
   - Mackenzie sells at least twice as many orange bulbs as red bulbs.

   **Given that Mackenzie earns $1 for every red bulb sold and $2 for every orange bulb sold, what is the maximum profit Mackenzie can earn each day?**



8) A company sells two different products. The first item is soap. The second item is lotion. Information about the sales of both items is below.

**Soap**
The company sells soap in two varieties: bars of soap and bottles of liquid soap. The soap sales must fit within the constraints given below:

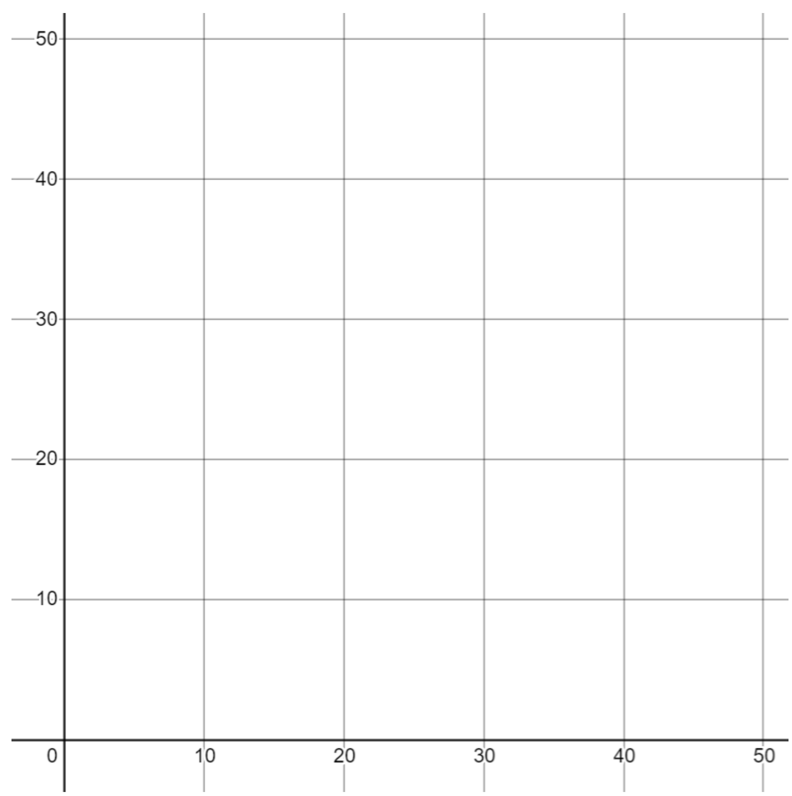
- The company sells a maximum of 40 soaps per day.
- At least 5 bars of soap are sold each day.
- No fewer than 5 bottles of liquid soap are sold each day.
- The company sells no more than 3 times as many bars of soap as bottles of liquid soap

The company earns a profit of $2 per bar of soap and $3 per bottle of liquid soap.

**Lotion**
The company also sells two types of lotion: hand lotion and body lotion. Every day they sell 20 bottles of hand lotion and 25 bottles of body lotion.

The company earns the same profit on the total sale of lotion as the maximum profit from the sale of soap.

**Given that they earn a profit of $3 on each bottle of hand lotion, what is the profit earned on each bottle of body lotion?**



Optimization Unit – Exam Style Questions

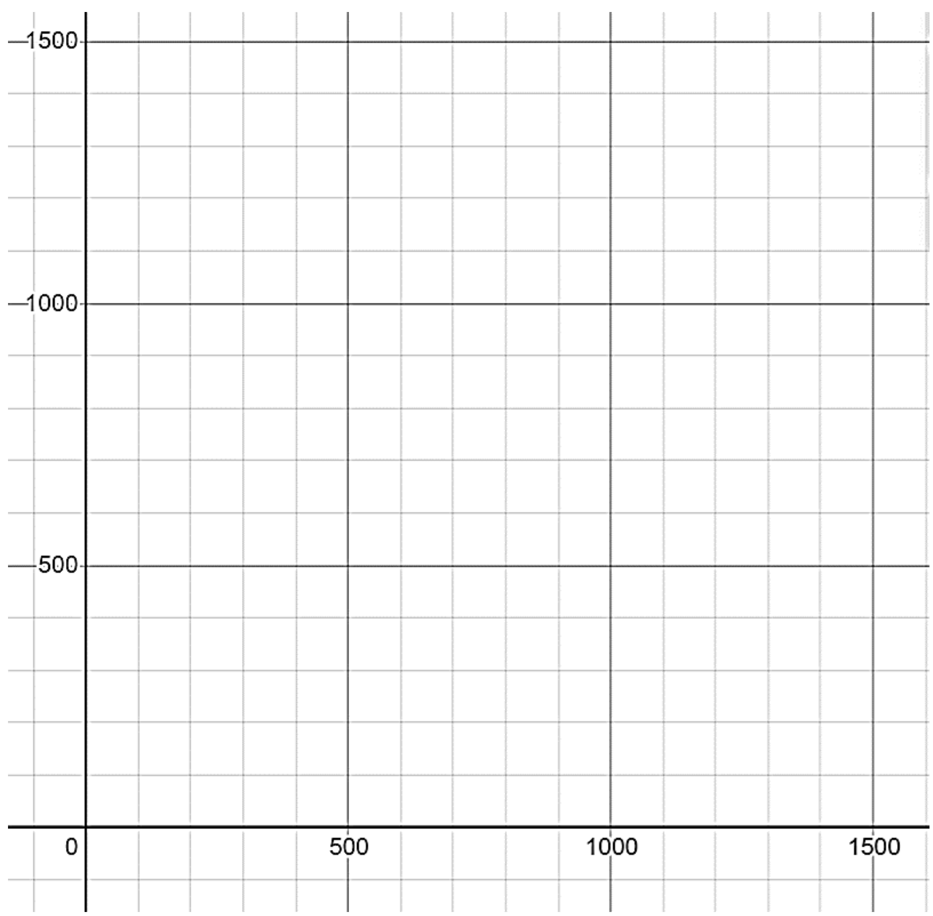
9) A company owns a vehicle manufacturing plant which has two assembly lines. One of the lines makes cars and one of the lines makes trucks. On a typical day, the plant must adhere to the following constraints in manufacturing cars and trucks.

- The plant can make no more than 1200 vehicles per day.
- The assembly line making cars can produce a minimum of 200 cars per day.
- The assembly line making trucks can produce no more than 800 cars per day.
- The manufacturing plant makes no more than twice times as many cars as trucks.

The company makes a profit of $2000 per car and $2500 per truck.

Today the manufacturing plant experienced a breakdown in the assembly lines and as a result the production capacity has been reduced. Today, the plant can make no more than 900 vehicles.

**By how much did the breakdown reduce the maximum profit when compared to a typical day?**



Optimization Unit – Exam Style Questions