Population and Financial Math Unit – Exponents Review

3.1 EXPONENTS REVIEW

An exponent is a number (or variable) that tells us how many times to multiply the base by itself.

$$y = \underset{\nwarrow\ \text{base}}{a}^{x \leftarrow \text{exponent}}$$

$3^5 = 3 \times 3 \times 3 \times 3 \times 3$

$x^5 = x \times x \times x \times x \times x$

Ex: Calculate each of the following expressions.

$7^3 =$

$(0.25)^8 =$

$8(1.025)^3 =$

$1000(0.95)^7 =$

Practice Questions

1) Calculate each of the following expressions.

a) $12^3 =$ 343

b) $(0.87)^{12} =$ 0.1880

c) $500(1.6)^2 =$ 1280

d) $2000(0.82)^{15} =$ 101.9149

Population and Financial Math Unit – Population and Compound Interest

3.2 POPULATION AND COMPOUND INTEREST

We can use an exponential function for questions about population or compound interest.

$$C_n = a(rate)^n$$

Where: a is the initial value (starting amount)

C_n is the final value (ending amount)

rate is how fast something is increasing or decreasing

n is the time

*We always have to make sure that time is measured in the same units as the rate of growth/decay.

We can determine rate as follows:

If we are given words: “doubles”, $c = 2$. “triples”, $c = 3$. “half”, $c = 1/2$ (or 0.5), etc.

If we are given a percent:

If increase: $rate = 1 + \frac{\%}{100}$

If decrease: $rate = 1 - \frac{\%}{100}$

Ex: An initial population of 1000 bacteria increases at a rate of 3% per day. What is the population after 4 days?

a: 1000

C_n: ?

rate: $1 + \frac{3}{100} = 1.03$

n: 4

$C_n = 1000(1.03)^4$

$C_n = 1125.51$

∴ There will be 1126 bacteria in 4 days

Ex: An initial population of 800 fish decreases by half every 6 months. What will the population be in 4 years?

a: 800

C_n: ?

rate: 1/2 or 0.5

* n: 8

It decreases every 6 months, so it will decrease 8 times in 4 years

$C_n = 800(0.5)^8$

$C_n = 3.125$

∴ There will be 3 fish in 4 years

Population and Financial Math Unit – Population and Compound Interest

Ex: A city has a population of 30 000 people. The population is declining at a rate of 12% per year. What will the population be after 7 years?

a: 30 000

C_n:

rate: $1 - \frac{12}{100} = 0.88$

n: 7

$C_n = 30\,000(0.88)^7$

$C_n = 12\,260.27$

$\therefore$ There will be 12 260 People after 7 years.

Ex: Miguel invests $500 at an annual compound interest rate of 2.6%. How much is Miguel's investment worth after 8 years?

a: 500

C_n:

rate: $1 + \frac{2.6}{100} = 1{,}026$

n: 8

$C_n = 500(1{,}026)^8$

$C_n = 613.97$

$\therefore$ Miguel will have \$613.97 after 8 years.

Ex: You borrow $700 at a monthly compound interest rate of 1.5%. How much will you have to repay after 2 years?

a: 700

C_n:

rate: $1 + \frac{1{,}5}{100} = 1{,}015$

n: 24

interest applied every month, so 24 times in 2 years.

$C_n = 700(1{,}015)^{24}$

$C_n = 1000.65$

$\therefore$ You will have to repay \$1000.65 after 2 years.

Practice Questions

1) 140 students attend an art camp. It is expected that this number will increase at a rate of 4% every year. What will the population be after 5 years?

2) In an aquatic environment, there are 480 different species. It is expected that the number of species will decrease by 10% every year. How many species will there be after 8 years?

3) There are 4 bacteria in a petri dish. The population doubles every 20 minutes. What will the population be after 4 hours?

4) $5000 is invested at an interest rate of 6% compounded annually. What is the value of this investment in 5 years?

5) You purchase a new computer for $1200. The value of the computer depreciates at a rate of 43% annually. How much will your computer be worth after 3 years?

6) Valerie invests $2500 at a quarterly compound interest rate of 2.3%. What is the value of her investment after 6 years?

3.3 SOLVING FOR OTHER VARIABLES

If we need to solve for either a or c, we can plug in the variables we know and use algebra to solve, or we can use the following formulas.

$$a = \frac{C_n}{rate^n} \quad \text{and} \quad rate = \sqrt[n]{\frac{C_n}{a}}$$

Note: When solving for percent, remember $rate = 1 \pm \frac{\%}{100}$, so you need to do a few more calculations to determine the percent.

Ex: It is expected that the value of a stock will triple every 6 months. After 1 year, the value of the stock is $10.80. What was the initial value of the stock?

a :
C_n : 10.80
rate : 3
n : 2

$C_n = a(rate)^n$
$10.80 = a(3)^2$
$10.80 = a(9)$
$\div 9 \quad \div 9$
$1.20 = a$

OR

$a = \frac{C_n}{rate^n}$
$a = \frac{10.80}{3^2}$
$a = 1.20$

∴ The initial value was $1.20

Ex: A grocery basket costs $240 in 2018. If the cost of the same basket will be $269.97 in 2021, what was the annual inflation rate during this time?

a : 240
C_n : 269.97
rate : ?
n : 3

$C_n = a(rate)^n$
$269.97 = 240(rate)^3$
$\div 240 \quad \div 240$
$1.1249 = (rate)^3$
$\sqrt[3]{1.1249} = rate$
$1.0400 = rate$
-1
0.04
$\times 100 = 4\%$

To get %,
find rate,
then
−1
×100

OR $rate = \sqrt[n]{\frac{C_n}{a}}$
$rate = \sqrt[3]{\left(\frac{269.97}{240}\right)}$
$rate = 1.0400$
-1
0.0400
$\times 100$
4%

∴ The annual inflation rate was 4%.

Practice Questions

1) A student posts a photograph online. On the first day the photo is posted, it is seen by 38 people. After 10 days, the photo has been seen by 2191. What is the rate by which the number of people seeing the photo increases each day?

2) Calcium chloride is used to de-ice roads during Quebec's winters. During an ice storm, 7.5 cm of ice accumulates on the roads because the de-icer spreads the salt. After 3 hours the thickness of the ice will be 5.11 cm? By what rate does the salt reduce the thickness of ice per hour?

3) Super Balls are made out of a synthetic rubber. They bounce much higher than other balls. At each bounce, a Super Ball loses 8% of its maximum height from the previous bounce. After 8 bounces, the ball has a height of 200 cm. From what height was the ball originally dropped?

4) After 8 years, the accumulated capital of an investment at an annual compound interest rate of 2% will be $7029.96. What was the amount of the initial investment?

5) Money is invested at an interest rate of 4% compounded annually. If the future value of this investment after 6 years is $3796, what is the value of the initial investment?

6) $4700 was invested and yields $8699.37 after 8 years. What was the annual compound interest rate?

7) $2800 was invested and after 2 years the value of the investment was $3087. What was the annual compound interest rate?

8) At what annual compound interest rate should we invest so that our initial investment will be doubled after 10 years?

3.4 LOGARITHM BASICS

We have solved exponential functions for every variable except x. In order to solve for x, we need to use logarithms.

A logarithm is a function (and a button on your calculator, just like the trig functions: sine, cosine, and tangent).

We can re-write an exponential function as a logarithmic function.

$$y = c^x \Leftrightarrow x = \log_c y$$

Ex: Write the following expressing using logs

a) $4 = 5^x$

$x = \log_5 4$

b) $150 = 3^x$

$x = \log_3 150$

c) $6561 = 3^8$

$8 = \log_3 6561$

To solve for x, we can use the following formula

$$x = \frac{\log of\ the\ lonely\ number}{\log of\ the\ number\ with\ the\ exponent}$$

Ex: Solve for x

$8 = 2^x$

$x = \frac{\log 8}{\log 2}$

$x = 3$

$150 = 4(1.05)^x$

$\div 4 \quad \div 4$

$37.5 = 1.05^x$

$x = \frac{\log 37.5}{\log 1.05} = 74.2843$

Practice Questions

1) Solve each of the following for x.

a) $20 = 3^x$

b) $1.79 = 2.5^x$

c) $120.5 = 1.03^x$

3.5 USING LOGS TO SOLVE POPULATION AND FINANCIAL QUESTIONS

In order to solve an exponential function for x you can either plug in what you know and use algebra (remembering to solve using logs) or you can use the following formula:

$$n = \frac{\log\left(\frac{C_n}{a}\right)}{\log(rate)}$$

Ex: Bill has 2g of cells in a petri dish. The amount of cells increases by 12% every month. How long does it take for Bill to have 4g of cells?

$a: 2$

$C_n: 4$

rate $1+\frac{12}{100} = 1.12$

$n: ?$

$C_n = a(rate)^n$

$4 = 2(1.12)^n$

$\div 2 \quad \div 2$

$2 = 1.12^n$

$n = \frac{\log 2}{\log 1.12}$

$n = 6.1163$

or

$n = \frac{\log\left(\frac{4}{2}\right)}{\log(1.12)}$

$n = 6.1163$

∴ It will take 6.1163 months for Bill to have 4g of cells.

Ex: An initial investment of $3000 earns an annual compound interest rate of 2%. How long will it take for the investment to be worth $3312.24?

$a: 3000$

$C_n: 3312.24$

rate $1+\frac{2}{100} = 1.02$

$n: ?$

$C_n = a(rate)^n$

$3312.24 = 3000(1.02)^n$

$\div 3000 \quad \div 3000$

$1.1041 = 1.02^n$

$n = \frac{\log 1.1041}{\log 1.02}$

$n = 5.0009$

$n = 5$

or

$n = \frac{\log\left(\frac{C_n}{a}\right)}{\log(rate)}$

$n = \frac{\log\left(\frac{3312.24}{3000}\right)}{\log 1.02}$

$n = 5$

∴ It will take 5 years.

Practice Questions

1) $2500 is invested at an annual compound interest rate of 3%. How long will it take for the investment to be worth $2985?

2) $5000 is invested at an annual interest rate of 2% compounded annually. How long will it take for the investment to be worth $5520.40?

3) A petri dish contains 150 bacteria. The number of bacteria increases by 20% every hour. How long will it take for there to be 500 bacteria?

4) Water from a tank evaporates at a rate of 1% of its volume every hour. After how many hours will the tank hold 4184 L of water, if the tank initially contained 4650 L of water?

Population and Financial Math Unit – Simple Interest

3.6 SIMPLE INTEREST

When working with financial math (ex: finding the future value of an investment, etc.), the question may ask about simple interest or compound interest.

Simple interest means you only earn interest on your initial investment. For example, if you invest \$500 at an annual interest rate of 1%, you would earn 1% of \$500 (or \$5) for each year you kept your money in the investment.

We do not need exponential functions or logs to calculate investments with simple interest, but instead use the formula:

$$C_n = a(1 + rate * n)$$

Where:

C_n is the final amount

a is the initial investment

rate is the interest rate $\left(\frac{\%}{100}\right)$

n is the time (if necessary, transform t so it is in the same units as the interest rate)

Ex: Jill invests \$5000 at a simple interest rate of 6% annually. How much money does Jill have after 5 years?

a : 5000

C_n : ?

rate $\frac{6}{100} = 0,06$

n : 5

$C_n = a(1 + rate \times n)$

$C_n = 5000(1 + \underline{0,06 \times 5})$

$C_n = 5000(\underline{1 + 0.3})$

$C_n = \underline{5000(1,3)}$

$C_n = 6500$

∴ Jill has \$6500 after 5 years.

Ex: Ryan invests \$1000 at an annual simple interest rate of 4%. How much money does Ryan have after 18 months?

a : 1000

C_n : ?

rate $\frac{4}{100} = 0,04$

n : 1,5

18 months is 1,5 years

$C_n = a(1 + rate \times n)$

$C_n = 1000(1 + \underline{0,04 \times 1,5})$

$C_n = 1000(\underline{1 + 0.6})$

$C_n = \underline{1000(1,6)}$

$C_n = 1600$

∴ Ryan has \$1600.

Practice Questions

1) Determine the accumulated capital (the final amount of the investment) if you invested $8000 for 5 years at an annual simple interest rate of 7%.

2) Carlos invests $3600 at a monthly simple interest rate of 0.45%. What is the value of Carlos' investment after 7 years?

3) Mikka is saving money to buy a car. She wants to have at least $5000 for the down payment and so she makes two investments.

Investment 1: Mikka invests $2247.19 at a monthly simple interest rate of 1.3% for 3 years.
Investment 2: Mikka invests $1200 at a weekly simple interest rate of 0.3% for 3 years.

After 3 years, will Mikka have reached her goal?

*NOTE: THE EQUATIONS BELOW ARE WRITTEN WITH DIFFERENT VARIABLES IN YOUR WORKBOOK. THEY HAVE BEEN CORRECTED IN THIS VERSION

3.7 SOLVING FOR OTHER VARIABLES

We will not always be asked to solve for the accumulated capital (final value) of an investment. We can solve for the other variables by plugging in what we know and using algebra to solve, or by using the following formulas:

$$a = \frac{C_n}{1 + rate * n} \qquad n = \frac{\frac{C_n}{a} - 1}{rate} \qquad rate = \frac{\frac{C_n}{a} - 1}{n}$$

Ex: Rasha invested some money for 11 years at an annual simple interest rate of 7.4%. After 11 years, the investment was work $7890.90. How much did Rasha invest initially?

a : ?
C_n : 7890.90
rate: $\frac{7.4}{100} = 0.074$
n : 11

$C_n = a(1 + rate \times n)$
$7890.90 = a(1 + 0.074 \times 11)$
$7890.90 = a(1.814)$
$\div 1.814 \quad \div 1.814$
$a = 4350$

OR

$a = \frac{C_n}{1 + rate \times n}$
$a = \frac{7890.90}{1 + 0.074 \times 11}$
$a = 4350$

∴ Rasha invested $4350

Ex: How long will it take an initial capital of $9300 to earn an accumulated capital of $14 322 at an annual simple interest rate of 8%?

a : 9300
C_n 14322
rate $\frac{8}{100} = 0.08$
n ?

$C_n = a(1 + rate \times n)$
$14322 = 9300(1 + 0.08n)$
$\div 9300 \quad \div 9300$
$1.54 = 1 + 0.08n$
$-1 \quad -1$
$0.54 = 0.08n$
$\div 0.08 \quad \div 0.08$
$6.75 = n$

OR

$n = \frac{\left(\frac{C_n}{a} - 1\right)}{rate}$
$n = \frac{\left(\frac{14322}{9300} - 1\right)}{0.08}$
$n = 6.75$

∴ It will take 6.75 years

Ex: What is the annual simple interest rate for an investment of $2400 that yields $4646.40 after 9 years.

a : 2400
C_n : 4646.40
rate: ?
n : 9

Multiply rate by 100 to get %

$C_n = a(1 + rate \times n)$
$4696.40 = 2400(1 + rate \times 9)$
$\div 2400 \quad \div 2400$
$1.936 = 1 + rate \times 9$
$-1 \quad -1$
$0.936 = rate \times 9$
$\div 9 \quad \div 9$
$0.104 = rate$
$\times 100$
10.4%

OR

$rate = \frac{\left(\frac{C_n}{a} - 1\right)}{n}$
$rate = \frac{\left(\frac{4696.40}{2400} - 1\right)}{9}$
$rate = 0.104$
$\times 100$
$= 10.4\%$

∴ Need an annual simple interest rate of 10.4%

Population and Financial Math Unit – Solving for Other Variables

Practice Questions

1) After 7.5 years, an investment is worth $4199.75. What was the amount of the initial investment given that the simple interest rate was 3.8% per half-year?

2) $1800 is invested at a monthly simple interest rate of 2.25%. How long does it take for the investment to be worth $3420?

3) $9000 is invested for 7 years and 4 months. It is now worth $18 504. What was the monthly simple interest rate?

4) You borrowed \$3700 at a daily simple interest rate of 0.02%. When you repaid the loan, you paid \$4780.40. How long was it before you repaid the loan?

5) After 2.5 years, the repayment of a debt will be \$11 770.75 at a weekly simple interest rate of 0.15%. How much money was borrowed initially?

6) The repayment of a \$7 350 loan is \$14 891.10 after 9 years. Determine the quarterly simple interest rate.

3.8 POPULATION AND FINANCIAL MATH EXAM STYLE QUESTIONS

Multiple Choice

1) Keana initially invested \$5000. She would like to know exactly when her investment will be doubled. She uses the expression below.

$$C_n = 5000(1.03)^n$$

where C_n is the future value and n is the number of years.

Which of the following equations would be used to calculate the exact amount of time it will take for her investment to double?

A) $n = \frac{\log 1.03}{\log 2}$

C) $n = \frac{\log 10\,000}{\log 2}$

B) $n = \frac{\log 10\,000}{\log 5000}$

D) $n = \frac{\log 2}{\log 1.03}$

Short Answer

2) Tarek invested an amount of money 5 years ago at an interest rate of 3% compounded every six months.

The value of Tarek's investment can be determined by using the following rule.

$$C_n = a(1.03)^n$$

where n: number of 6-month periods elapsed since the beginning of the investment

C_n: value of the future investment, in dollars (\$)

Today, the value of Tarek's investment is \$1612.70.

To the nearest dollar, what was the value of Tarek's initial investment?