Review Unit – Solving Algebraic Equations

R.1 SOLVING ALGEBRAIC EQUATIONS

Variables are letters or symbols that represent numbers we do not know or that can change. We can use algebra to find the exact value of the variable.

There are two keys to solving equations:

1) To get rid of a number, you do its opposite (for example, if it was addition, use subtraction).
2) Whatever you do to one side, you do the same to the other side.

Let's look at 5 different levels.

Level 1: Variable on one side with only addition or subtraction

Get rid of the addition or subtraction by doing the opposite on both sides.

Ex: Solve for x.

a) $x + 2 = 3$

$-2 \quad -2$

$x = 1$

b) $x - 4 = 8$

$+4 \quad +4$

$x = 12$

c) $4x = 20$

$\div 4 \quad \div 4$

$x = 5$

d) $\frac{x}{10} = 3$

$\times 10 \quad \times 10$

$x = 30$

Practice Questions

1) Solve for x:

a) $x - 6 = 10$

b) $-8 - x = 11$

c) $45 = 5x$

d) $18 = \frac{x}{4}$

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Level 2: Variable on one side with addition or subtraction and multiplication.

Get rid of the addition or subtraction first, then get rid of the multiplication or division.

Ex: Solve for x.

a) $3x + 9 = 12$

$-9 \quad -9$

$3x = 3$

$\div 3 \quad \div 3$

$x = 1$

b) $4 + 2x = 10$

$-4 \quad -4$

$2x = 6$

$\div 2 \quad \div 2$

$x = 3$

c) $\frac{x}{3} - 25 = 75$

$+25 \quad +25$

$\frac{x}{3} = 100$

$\times 3 \quad \times 3$

$x = 300$

d) $-7 + \frac{x}{5} = 20$

$+7 \quad +7$

$\frac{x}{5} = 27$

$\times 5 \quad \times 5$

$x = 135$

Practice Questions

2) Solve for x:

a) $4x - 6 = 6$

b) $8 + \frac{x}{7} = 43$

c) $15 = -5 + \frac{x}{2}$

d) $33 = 10x + 3$

Review Unit – Solving Algebraic Equations

Level 3: Variables and addition or subtraction on both sides

Get rid of the variable on one side, then get rid of the addition or subtraction on the other side.

Ex: Solve for x.

a) $3x + 3 = 13 - 2x$

$-3 \quad -3$

$3x = 10 - 2x$

$+2x \quad +2x$

$5x = 10$

$\div 5 \quad \div 5$

$x = 2$

b) $4 - 4x = 18 - 6x$

$-4 \quad -4$

$-4x = 14 - 6x$

$+6x \quad +6x$

$2x = 14$

$\div 2 \quad \div 2$

$x = 7$

c) $28x - 15 = 8x + 65$

$+15 \quad +15$

$28x = 8x + 80$

$-8x \quad -8x$

$20x = 80$

$\div 20 \quad \div 20$

$x = 4$

d) $150 + 2x = 300 - x$

$-150 \quad -150$

$2x = 150 - x$

$+x \quad +x$

$3x = 150$

$\div 3 \quad \div 3$

$x = 50$

Practice Questions

3) Solve for x:

a) $2x - 6 = 8 - 5x$

b) $8 + 7x = 3x + 28$

c) $15 - 2x = -5 + 2x$

d) $20x + 60 = -10x + 120$

Review Unit – Solving Algebraic Equations

Level 4: Brackets on one or both sides

Get rid of the brackets by expanding and then solve like in Level 3.

Ex: Solve for x.

a) $6(x-2) = 15 + 3x$

$6x - 12 = 15 + 3x$

$+12 \quad +12$

$6x = 27 + 3x$

$-3x \quad -3x$

$3x = 27$

$\div 3 \quad \div 3$

$x = 9$

b) $3(4+3x) = 4(2+x)$

$12 + 9x = 8 + 4x$

$-12 \quad -12$

$9x = -4 + 4x$

$-4x \quad -4x$

$5x = -4$

$\div 5 \quad \div 5$

$x = -4/5$

OR

$x = -0.8$

c) $2(7-8x) = 5(2x-4)$

$14 - 16x = 10x - 20$

$-14 \quad -14$

$-16x = 10x - 34$

$-10x \quad -10x$

$-26x = -34$

$\div -26 \quad \div -26$

$x = 17/13$

OR

$x = 1.3077$

Practice Questions

4) Solve for x:

a) $2(x+3) = 10 - 2x$

b) $7(2x+3) = 3(4x-2)$

c) $3(2-5x) = 5(4-6x)$

Review Unit – Solving Algebraic Equations

Level 5: Division (fractions) on one or both sides

Cross multiply then solve like Level 4.

Ex: Solve for x.

a) $\frac{3x+2}{4} = \frac{2-6x}{5}$

$5(3x+2) = 4(2-6x)$

$15x + 10 = 8 - 24x$

$-10 \quad -10$

$15x = -2 - 24x$

$+24x \quad +24x$

$39x = -2$

$\div 39 \quad \div 39$

$x = -\frac{2}{39}$

OR

$x = -0.0513$

b) $\frac{4-x}{3} = \frac{5-2x}{2}$

$2(4-x) = 3(5-2x)$

$8 - 2x = 15 - 6x$

$-8 \quad -8$

$-2x = 7 - 6x$

$+6x \quad +6x$

$4x = 7$

$\div 4 \quad \div 4$

$x = \frac{7}{4}$

OR

$x = 1.75$

c) $\frac{8x+3}{2} = 2 + 3x$

$8x + 3 = 2(2 + 3x)$

$8x + 3 = 4 + 6x$

$-3 \quad -3$

$8x = 1 + 6x$

$-6x \quad -6x$

$2x = 1$

$\div 2 \quad \div 2$

$x = \frac{1}{2}$

OR

$x = 0.5$

Practice Questions

5) Solve for x:

a) $\frac{2+4x}{3} = \frac{3x-1}{2}$

b) $\frac{2x+7}{2} = \frac{4-6x}{4}$

c) $8x + 3 = \frac{12x-5}{5}$

Practice Questions

6) $x + 3 = 5$

7) $2x + 4 = x + 12$

8) $3(x + 4) = 2(5 - 2x)$

9) $\frac{2x-3}{4} = \frac{2+x}{5}$

10) $3x - 6 = 18$

11) $x + 1 = 4(2 + 4x)$

12) $\frac{18x+3}{6} = 5x - 2$

13) $4x - 3 = 13$

Review Unit – Linear Equations

R.2 LINEAR EQUATIONS

Linear equations are generally written in the form $y = ax + b$

a is the slope (or rate of change)

b is the y-intercept (or initial value)

Identify the slope and y-intercept in the following equations

Ex: Find the slope and y-intercept of the following lines.

a) $y = 3x + 4$

slope: 3

y-intercept: 4

b) $y = \frac{2}{3}x - 7$

slope: $2/3$

y-intercept: -7

c) $y = x + 10$

slope: 1

y-intercept: 10

d) $y = -\frac{4}{3}x - 12$

slope: $-4/3$

y-intercept: -12

Sometimes we need to re-arrange the equation to get it in the form $y = ax + b$ before we can identify the slope and the y-intercept.

Ex: Find the slope and y-intercept of the following lines.

a) $2y = 8x + 4$

$\div 2 \quad \div 2 \quad \div 2$

$y = 4x + 2$

slope: 4

y-intercept: 2

b) $4y - 5x = 12$

$+5x \quad +5x$

$4y = 5x + 12$

$\div 4 \quad \div 4 \quad \div 4$

$y = \frac{5}{4}x + 3$

slope: $5/4$

y-intercept: 3

Practice Questions

1) Identify the slope and y-intercept of the following linear equation

a) $3x + 4y = 12$

b) $-2y - 10x + 4 = 0$

Review Unit – Linear Equations

Graphing a line

When we are graphing a line:

$$a = slope = \frac{rise}{run}$$

b = y-intercept (where the line crosses the y-axis).

Positive slope = "rise" up.
Negative slope = "rise" down.
Always run right.

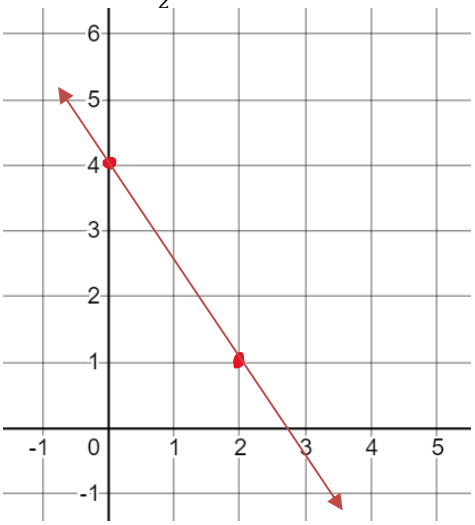
To graph a line:

1) Put a dot on the y-axis at b.
2) Starting from b, use slope to find a second point. Put a dot there.
3) Connect the dots using a ruler.
4) Draw arrows at each end of the line.

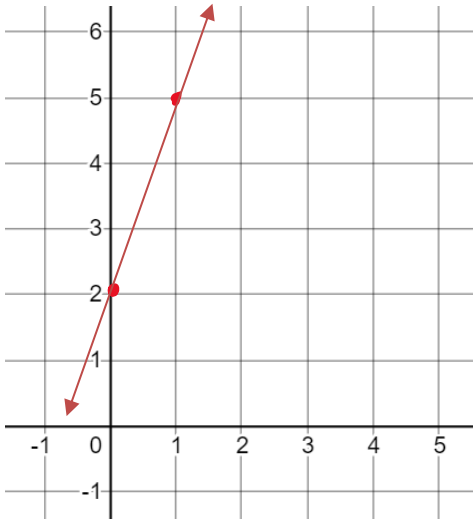
Remember:
If there is no b,
then $b = 0$

Ex: Graph the following lines.

a) $y = -\frac{3}{2}x + 4$



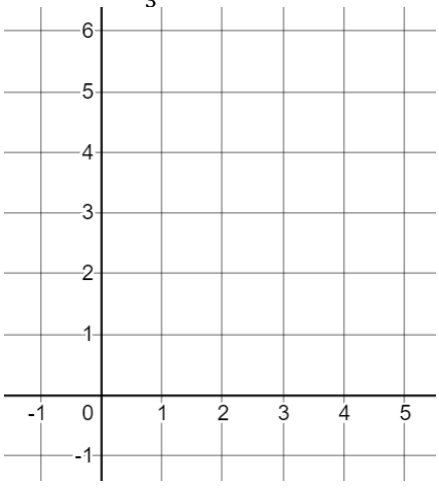
b) $y = 3x + 2$



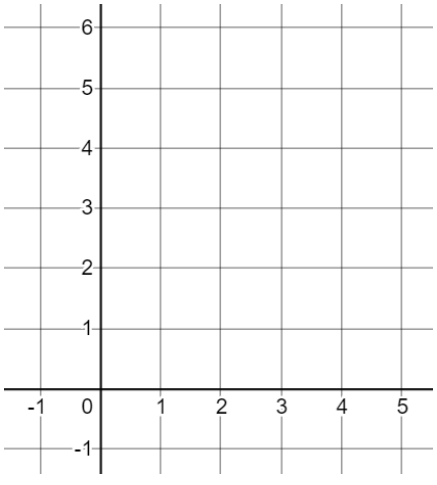
Practice Questions

2) Graph the following lines.

2a) $y = \frac{4}{3}x$



b) $y = -2x + 5$



Review Unit – Linear Equations

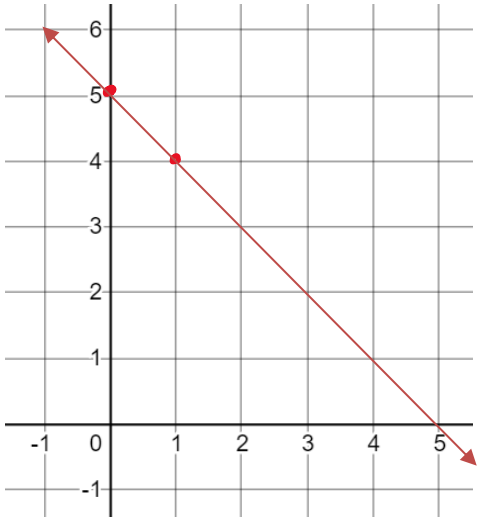
Sometimes we will have to re-arrange the equations before we can graph them.

Ex: Graph the following lines.

a) $x + y = 5$

$-x \quad -x$

$y = -x + 5$



b) $2x = 3y - 3$

$-3y \quad -3y$

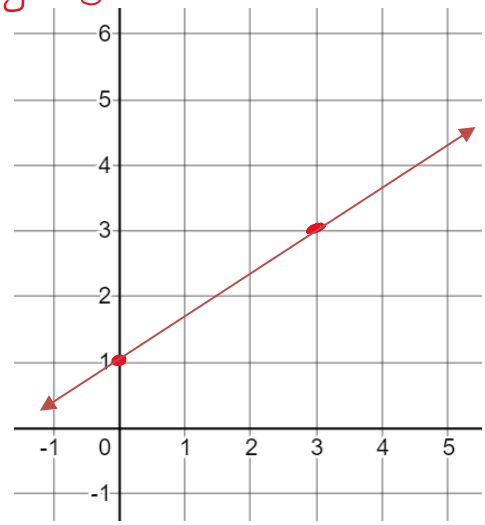
$-3y + 2x = -3$

$-2x \quad -2x$

$-3y = -2x - 3$

$\div -3 \quad \div -3 \quad \div -3$

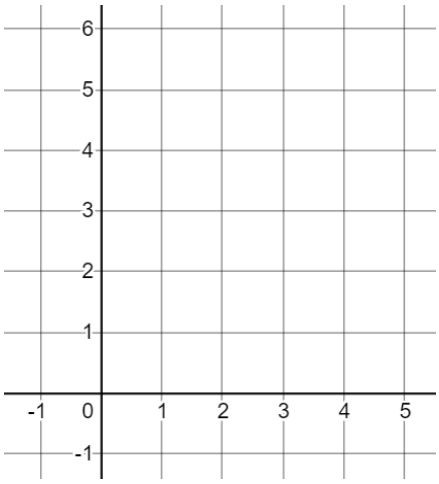
$y = \frac{2}{3}x + 1$



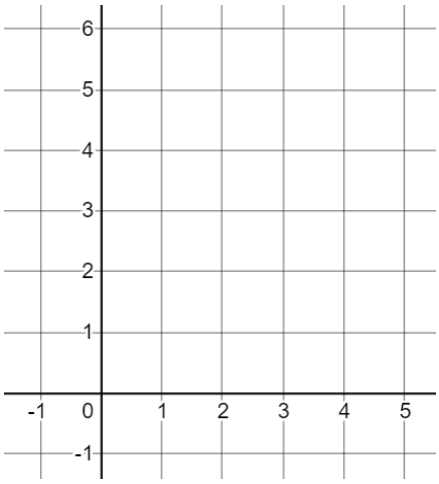
Practice Questions

3) Graph the following lines

a) $5x + 2y = 10$



b) $x + y = 3$



Review Unit – Linear Equations

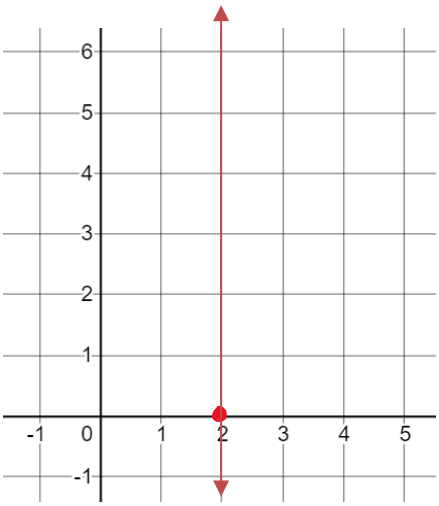
Sometimes we will have an equation of a line that only has one variable.

If the line only has an x (and not a y), put a dot on the x-axis at the number given and draw a vertical line.

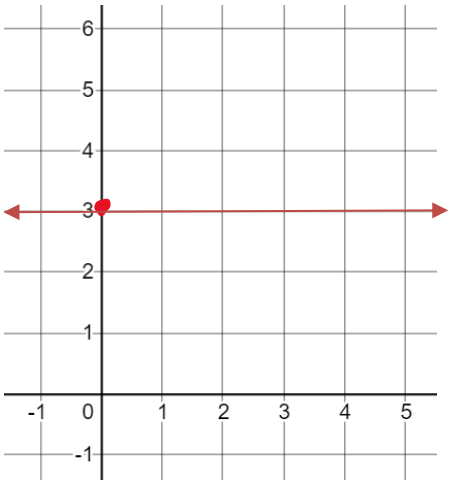
If the line only has a y (and not an x), put a dot on the y-axis at the number given and draw a horizontal line.

Ex: Graph the following lines.

a) $x = 2$



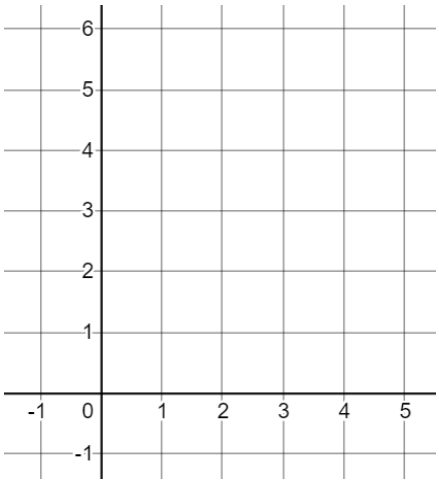
b) $y = 3$



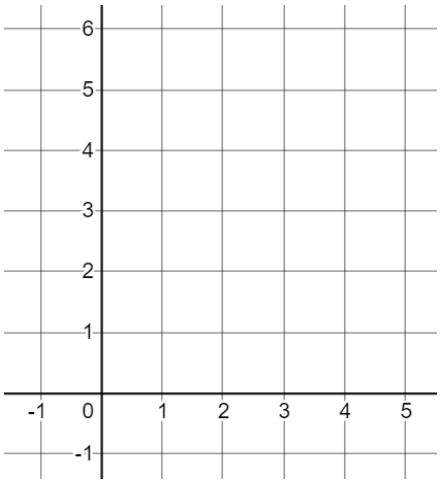
Practice Questions

4) Graph the following lines

a) $y = 5$



b) $x = 4$



Practice Questions

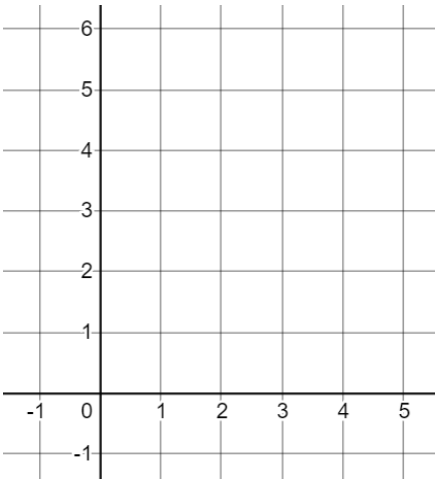
5) Find the slope and y-intercept of the line:

$$y = \frac{2}{3}x - 10$$

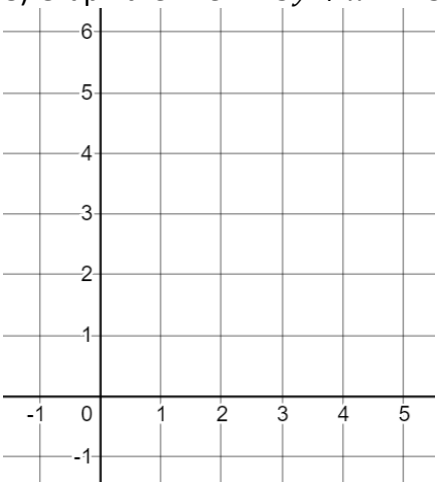
6) Find the slope and y-intercept of the line:

$3x - 7y = 35$

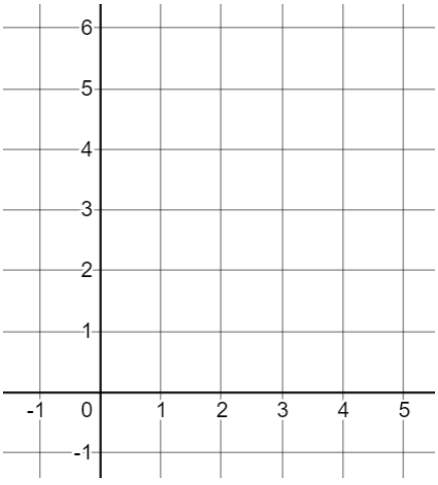
7) Graph the line: $y = 3x - 1$



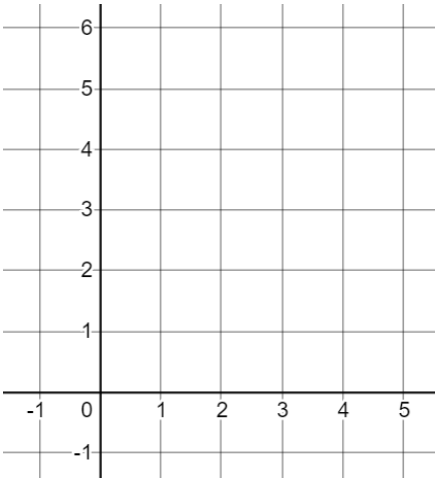
8) Graph the line: $3y + x = 15$



9) Graph the line: $x = 1$



10) Graph the line: $y = 4$



Review Unit – Linear Systems

R.3 LINEAR SYSTEMS

A linear system is when we have two lines.

The **solution** is the point (x, y) where the two lines cross each other. We can find the solution to a system of equations graphically or algebraically (using elimination, comparison, or substitution).

Using a graph to solve:

The find a solution, graph both lines. The solution is the point where the lines cross, written as an ordered pair (x, y). Remember, you may need to re-arrange the equation before you can graph it.

Ex: Find the solution to the linear system.

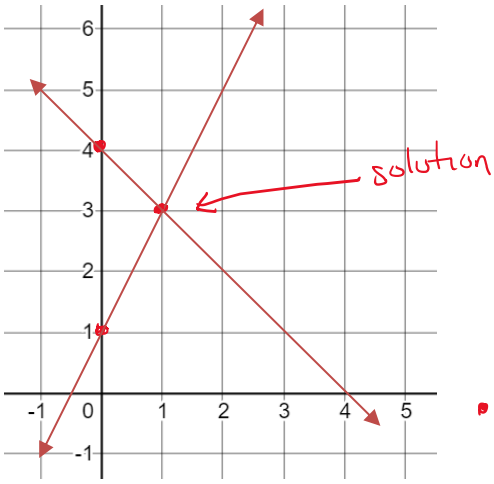
$y = 2x + 1$ and $x + y = 4$

$y = 2x + 1$

$x + y = 4$

$-x \quad -x$

$y = -x + 4$

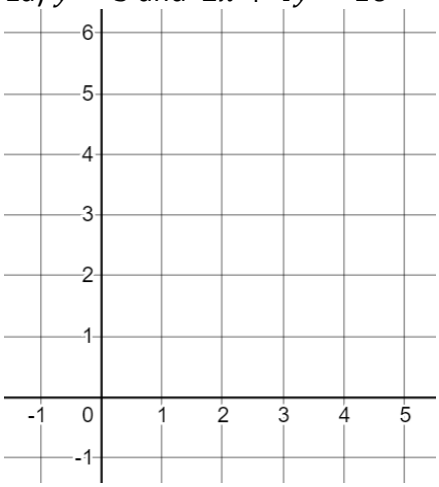


$\therefore$ the solution is $(1, 3)$

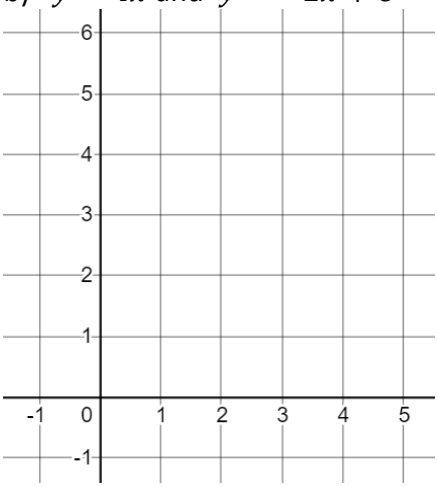
Practice Questions

1) Find the solution to the linear systems.

1a) $y = 3$ and $2x + 4y = 16$



b) $y = 4x$ and $y = -2x + 6$



Review Unit – Linear Systems

The graphing method is not an accurate way to solve linear systems. For example, it is difficult to tell the different between $(2.3, 4.6)$ and $(2.2, 4.7)$. Therefore, we will use algebra to solve linear systems.

There are 3 methods we can use: elimination, comparison, and substitution.

Using the elimination method to solve:

- Both lines must be in the form $Ax + By = C$.
- Multiply the entire first equation by the coefficient of x in the second equation.
- Multiply the entire second equation by the coefficient of x in the first equation, but change the sign.
- Add the two equations.
- Solve for the remaining variable.
- Use the solution in either equation to solve for the other variable.
- Write the solution (x, y).

Ex: Find the solution to the linear systems.

a) $2x + 5y = 16$ and $3x - 4y = 1$

$(2x+5y=16)\times 3 = 6x+15y=48$

$(3x-4y=1)\times -2 = -6x+8y=-2$

$23y = 46$

$\div 23 \quad \div 23$

$y = 2$

$2x+5y=16$

$2x+5(2)=16$

$2x+10=16$

$-10 \quad -10$

$2x = 6$

$\div 2 \quad \div 2$

$x = 3$

$\therefore$ solution is $(3, 2)$

b) $4x - 5y = 10$ and $y = -\frac{5}{3}x + 35$

$y = -\frac{5}{3}x + 35$

$+5/3x \quad +5/3x$

$\frac{5}{3}x + y = 35$

$(4x-5y=10)\times 5/3 = \frac{20}{3}x - \frac{25}{3}y = \frac{50}{3}$

$(\frac{5}{3}x+y=35)\times -4 = -\frac{20}{3}x - 4y = -140$

$-\frac{37}{3}y = -\frac{370}{3}$

$\div -\frac{37}{3} \quad \div -\frac{37}{3}$

$y = 10$

$4x-5y=10$

$4x-5(10)=10$

$4x-50=10$

$+50 \quad +50$

$4x = 60$

$\div 4 \quad \div 4$

$x = 15$

$\therefore$ solution is $(15, 10)$

Practice Questions

2) Find the solution to the linear systems.

a) $2x + 5y = -4$ and $3x - 2y = 13$

b) $3x + 4y = -6$ and $y = -2x + 1$

Review Unit – Linear Systems

Using the comparison method to solve:

- Both lines must be in the form $y = ax + b$.
- Take the $ax + b$ pieces from each equation and set them equal to each other $ax + b = ax + b$.
- Solve for x.
- Use either equation (and the value of x you just found) to solve for y.
- Write the solution (x, y).

Ex: Find the solution to the linear systems.

a) $y = 2x + 1$ and $y = -1.5x + 4.5$

$2x + 1 = -1.5x + 4.5$
$-1 \quad\quad -1$
$2x = -1.5x + 3.5$
$+1.5x \quad +1.5x$
$3.5x = 3.5$
$\div 3.5 \quad \div 3.5$
$x = 1$

$y = 2x + 1$
$y = 2(1) + 1$
$y = 2 + 1$
$y = 3$

∴ Solution $(1, 3)$

b) $y = -2x - 6$ and $5x + y = -3$

$-2x - 6 = -5x - 3$
$+6 \quad\quad +6$
$-2x = -5x + 3$
$+5x \quad +5x$
$3x = 3$
$\div 3 \quad \div 3$
$x = 1$

$y = -2x - 6$
$y = -2(1) - 6$
$y = -2 - 6$
$y = -8$

∴ Solution is $(1, -8)$

$5x + y = -3$
$-5x \quad -5x$
$y = -5x - 3$

Practice Questions

3) Find the solution to the linear systems.

a) $y = 2x + 5$ and $y = -4x + 11$

b) $y = 0.5x + 2$ and $y - 2x = -1$

Review Unit – Linear Systems

Using the substitution method to solve:

- This method works best if we already know the value of x or y.
- Use the equation that has both variables and replace the known variable.
- Solve for the missing variable.
- Write the solution as (x, y).

Ex: Find the solution to the linear systems.

a) $x = 2$ and $y = 3x + 8$

$y = 3x + 8$

$y = 3(2) + 8$

$y = 6 + 8$

$y = 14$

$\therefore$ solution $(2, 14)$

b) $y = 3$ and $3x + 4y = 20$

$3x + 4y = 20$

$3x + 4(3) = 20$

$3x + 12 = 20$

$-12 \quad -12$

$3x = 8$

$\div 3 \quad \div 3$

$x = 8/3$

$\therefore$ solution is $(8/3, 3)$

Practice Questions

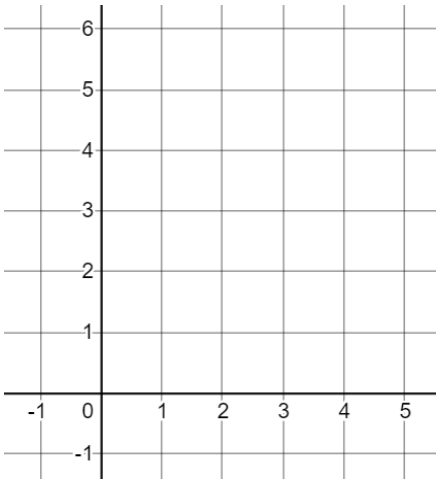
4) Find the solution to the linear systems.

a) $y = 5$ and $y = 2x - 15$

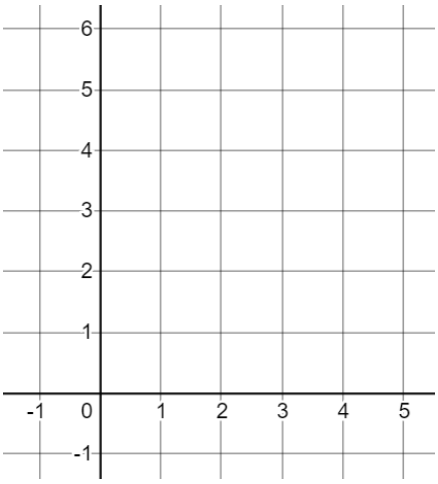
b) $x = 4$ and $3x + 2y = 20$

Practice Questions

6) Solve the system using graphing:
$y = 2x - 1$ and $y = x + 1$



7) Solve the system using graphing:
$y = -3x + 4$ and $y + 2 = 3x$



8) Solve the system using elimination:
$8x - 6y = -20$ and $-16x + 7y = 30$

9) Solve the system using elimination:
$-4y - 11x = 36$ and $20 = -10x - 10y$

Review Unit – Linear Systems

10) Solve the system using comparison:
$y = x - 13$ and $y = -2x + 5$

11) Solve the system using comparison:
$y = -4x + 2$ and $x - y = 3$

12) Solve the system using substitution:
$y = -5$ and $5x + 4y = -20$

13) Solve the system using substitution:
$x = 3$ and $4x - y = 20$