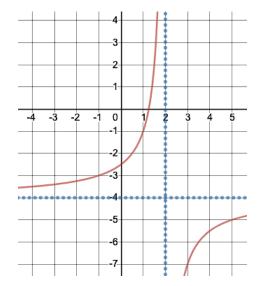
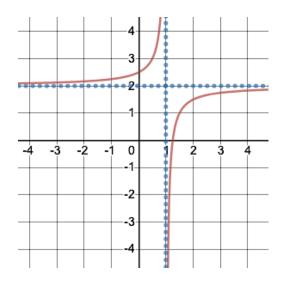
## **Rational Functions Review**

Sketch each of the following functions. Be sure to draw & label the asymptotes. 1.

a) 
$$f(x) = \frac{-3}{(x-2)} - 4$$



**b)** 
$$g(x) = \frac{4x-5}{2x-2}$$



Solve 2.

a) 
$$\frac{4}{-x+3} + 2 = -9$$

**b)** 
$$\frac{3}{2x+6} - 5 \ge 2$$

c) 
$$\frac{3x+8}{x-5} = 10$$

$$x = 3.36$$

$$x = 8.2857$$

Find the rule for the inverse of the following functions: 3.

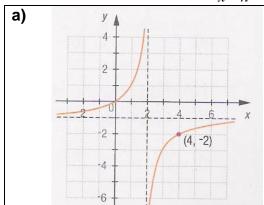
**a)** 
$$f(x) = \frac{2}{x+4} - 6$$

$$f(x) = \frac{2}{x+4} - 6$$
 
$$f^{-1}(x) = \frac{2}{x+6} - 4$$

**b)** 
$$g(x) = \frac{3x-6}{2x+4}$$

$$g^{-1}(x) = \frac{-4x - 6}{2x - 2}$$

Find the rule in the form  $f(x) = \frac{a}{x-h} + k$ 4.



$$f(x) = \frac{-2}{x-2} - 1$$

 $f(x) = \frac{-2}{x-3} + 4$ 

- f(x) is a rational function &  $f^{-1}(x)$  is its inverse. The following information is given b)
  - The domain is  $]-\infty,3[\cup]3,+\infty[$
  - The range is  $-\infty, 4 \cup 4, +\infty$
  - It passes through the point (2, 6)
  - Find the rule of f(x).
- Given the rational function  $f(x) = \frac{3x+5}{2x-1}$ . What are the equations of the asymptotes of this 5. function?

A) 
$$x = \frac{1}{2}$$
  $y = \frac{3}{2}$  C)  $x = \frac{3}{2}$   $y = \frac{1}{2}$ 

C) 
$$x = \frac{3}{2}$$
  $y = \frac{1}{2}$ 

B) 
$$x = \frac{1}{2}$$
  $y = -5$ 

$$x = \frac{1}{2}$$
  $y = -5$  D)  $x = -\frac{1}{2}$   $y = \frac{3}{2}$ 

A function is represented by the equation  $f(x) = \frac{-16}{x-15} + 12$ . What are the equations of the 6. asymptotes of  $f^{-1}(x)$ ?

A) 
$$x = 15$$
  
  $y = 12$ 

C) 
$$x = 12$$
  $y = 15$ 

D) 
$$x = 12$$
$$y = -15$$

Given the rational function  $f(x) = \frac{cx+d}{px+t}$  in which c>0, d>0, p>0 and t>0. What is the product of 7. the parameters/values that define its vertical and horizontal asymptotes?

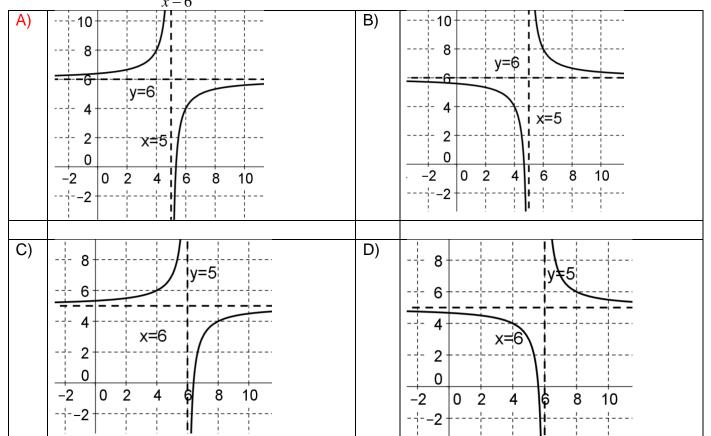
A) 
$$\frac{cd}{pt}$$

C) 
$$\frac{d}{p}$$

$$\mathbf{B)} \qquad \frac{-ct}{p^2}$$

D) 
$$\frac{td}{pc}$$

**8.** Given  $f(x) = \frac{5x - 32}{x - 6}$ , which graph below represents  $f^{-1}(x)$ ?



- **9.** The average profit, p, generated by a button sale varies according to the rule  $p(x) = \frac{2x 100}{x + 20}$  where x is the number of buttons sold.
  - a) What minimum number of buttons must be sold to turn a profit?

**b)** Keeping the context of the situation in mind, what is the maximum average profit generated by a button sold?

**10.** Why is the function  $f(x) = \frac{2x-4}{x-2}$  not a rational function?

Because it is a line (with a hole at x = 2).

11. During an experiment, 15mL of liquid A is initially mixed with 50mL of liquid B in order to generate a chemical reaction. Subsequently, 2mL of liquid A and 1mL of Liquid B are added to the mix every minute. A red coloring can be observed when the concentration of liquid A exceeds 50%. At what moment (i.e. time) is this reaction observed?

$$Conc A(\%) = \frac{Amount A}{Total Vol} \bullet 100$$

At 35 minutes.

- **12.** During an experiment on the crystallization of a solid from an aqueous solution, the solution's temperature and its concentration are made to vary. The information regarding this experiment is as follow:
  - The initial temperature of the solution is 20°C
  - The initial concentration of the solution is 1mol/L
  - The temperature decreases by 1°C/min
  - The concentration increases by 1.5mol/L/min

Crystals form when the ratio between  $\frac{concentration}{temperature}$  is 1.5mol/L/°C.

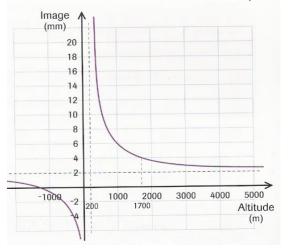
When will crystals begin to appear?

At 9.6667 minutes.

**13.** The manufacture of a new type of microchip involves an initial cost of \$500. A cost of \$2.25 for each additional chip is subsequently added to the total production cost. Consider a rule that yields the *production cost* of a chip according to the *number of manufactured chips*. What is the minimum number of chips that needs to be manufactured for the unit production cost to be less than \$3?

They must sell a minimum of 667 chips.

**14.** A small company specializes in aerial photography. During the summer its two employees fly over different cities and take aerial photos of certain properties. During the winter



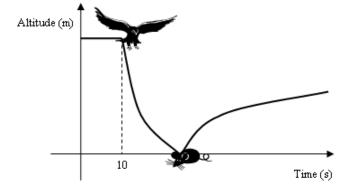
they contact the owners and offer to sell them the pictures of their land. Clearly, the higher the plane flies, the smaller the objects in the photo appear.

Using a camera, they photograph a round lake form several altitudes. They then measure the lake's diameter as it appears in the photographs. The graph below represents the data.

The diameter of the lake is 30mm in one of the photos. At what altitude was the photo taken?

307.1429 m

**15.** Sarah is conducting a field study on the eating habits of hawks. She watches one hawk in particular. She notices that after flying for 10 seconds, the hawk swoops down, catches its prey - a poor unsuspecting field mouse, and immediately takes off with it. The path of the



hawk's decent has been determined to follow a rational function and the path of its ascent follows a square root function. The point of ascent corresponds to the vertex of the square root function. Four seconds after the hawk catches the mouse, it is 8m above the ground.

The rational function is

$$f(x) = \frac{4}{x-10} - 2$$
, for  $x > 10$ .

How many meters above the ground will the hawk be 9 seconds after it catches its prey?

12 m