

Name Key Group \_\_\_\_\_

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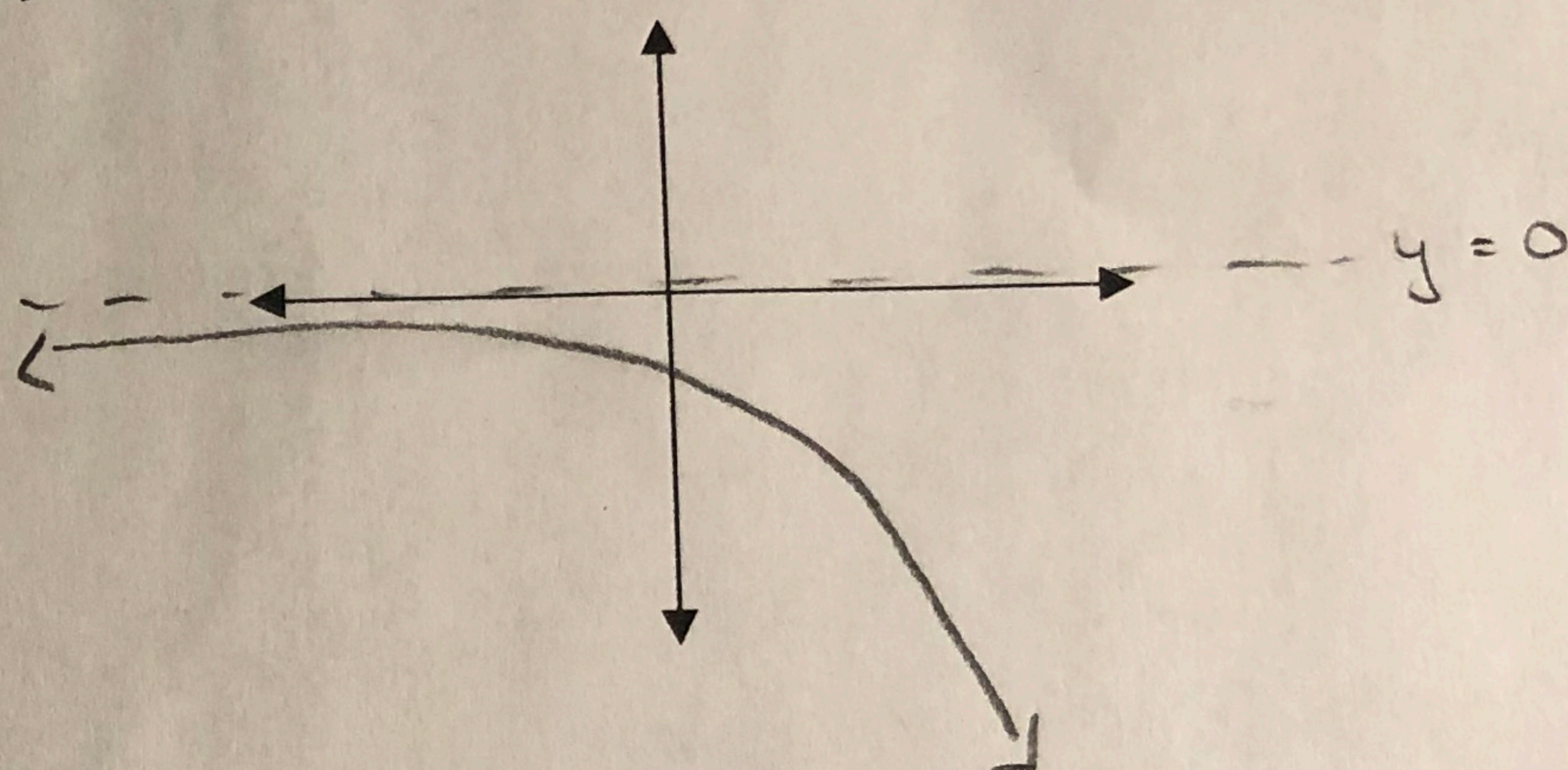
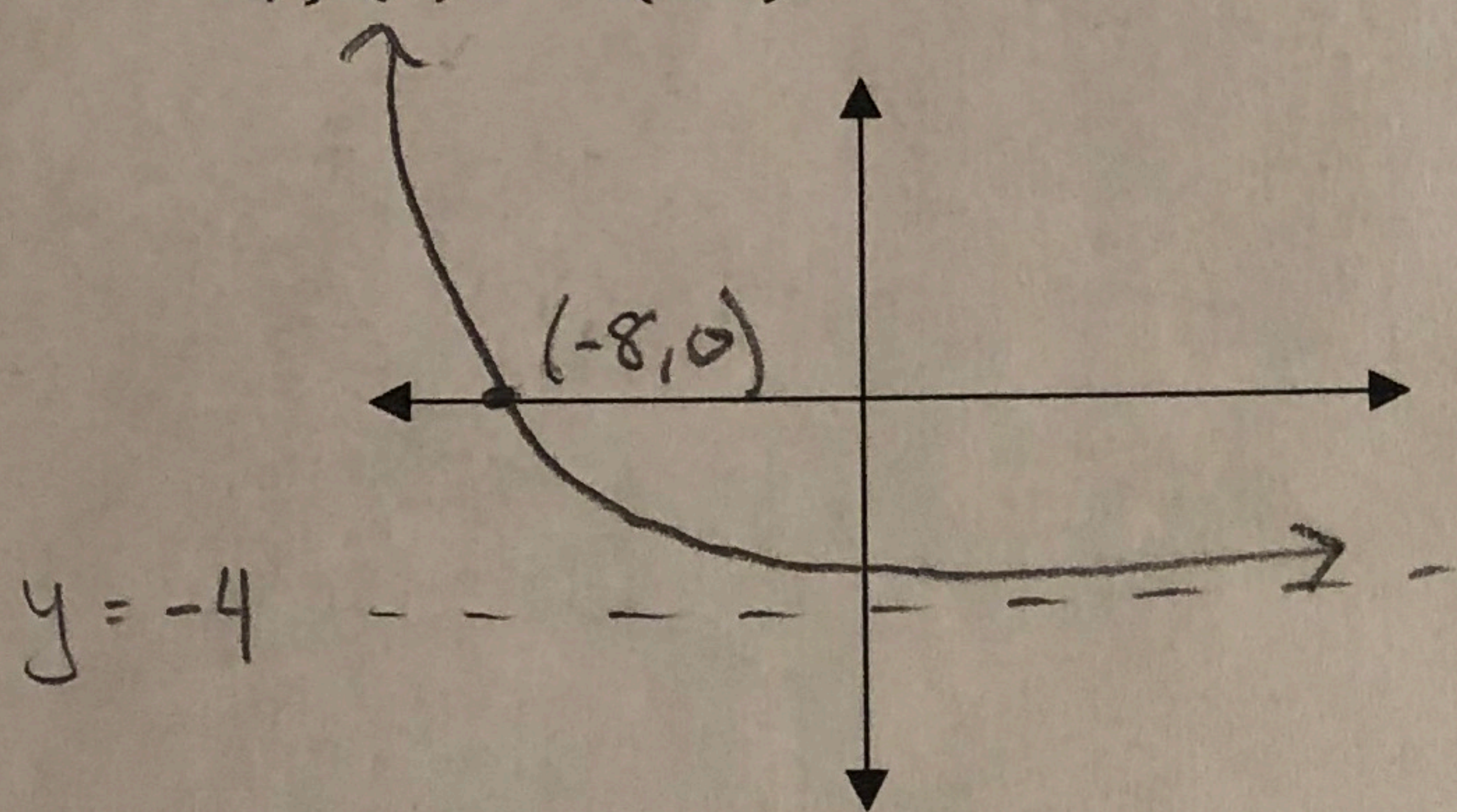
### Exponential Functions

1) Sketch the following functions

(4 pts)

a)  $f(x) = 4(0.5)^{x+8} - 4$

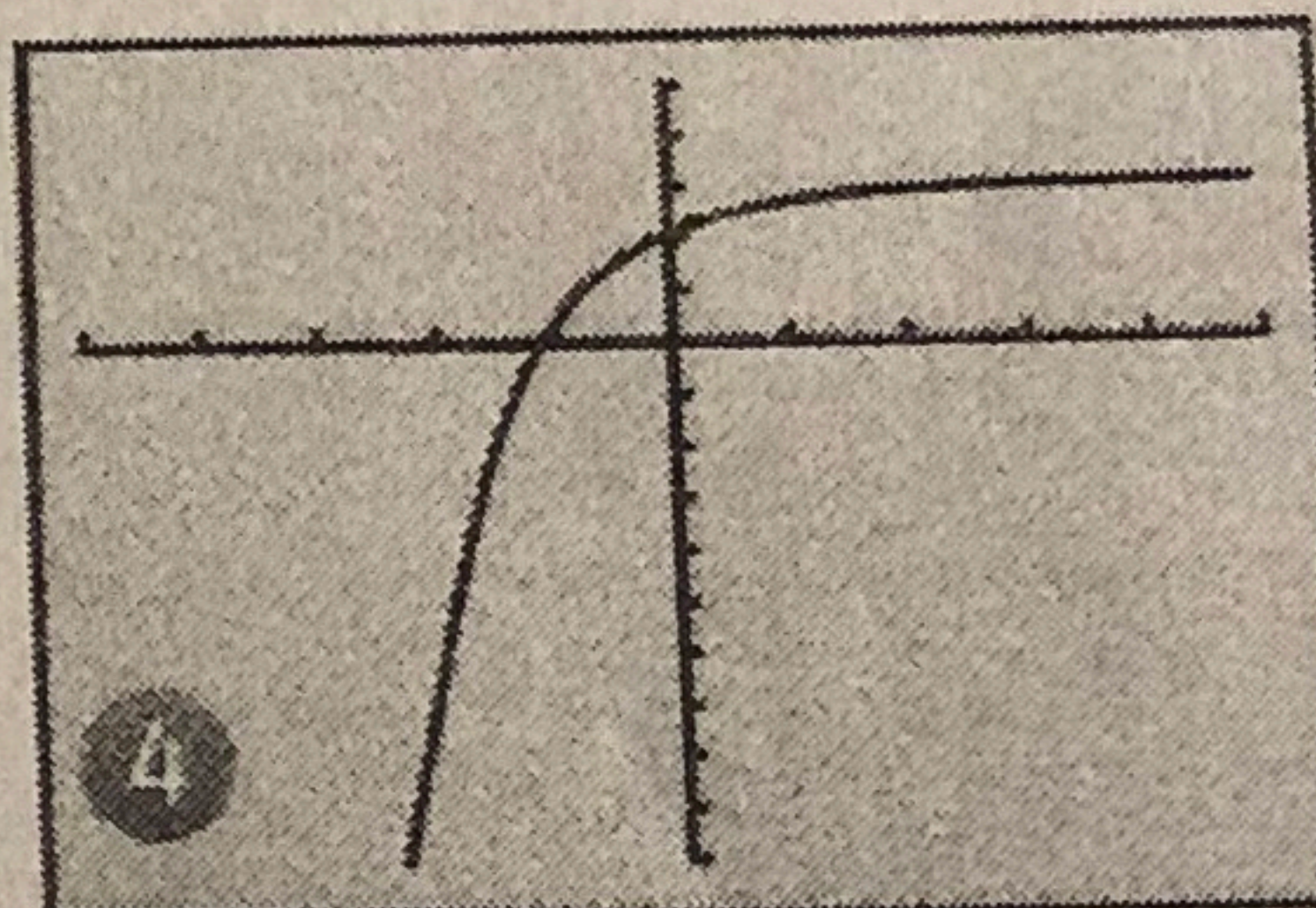
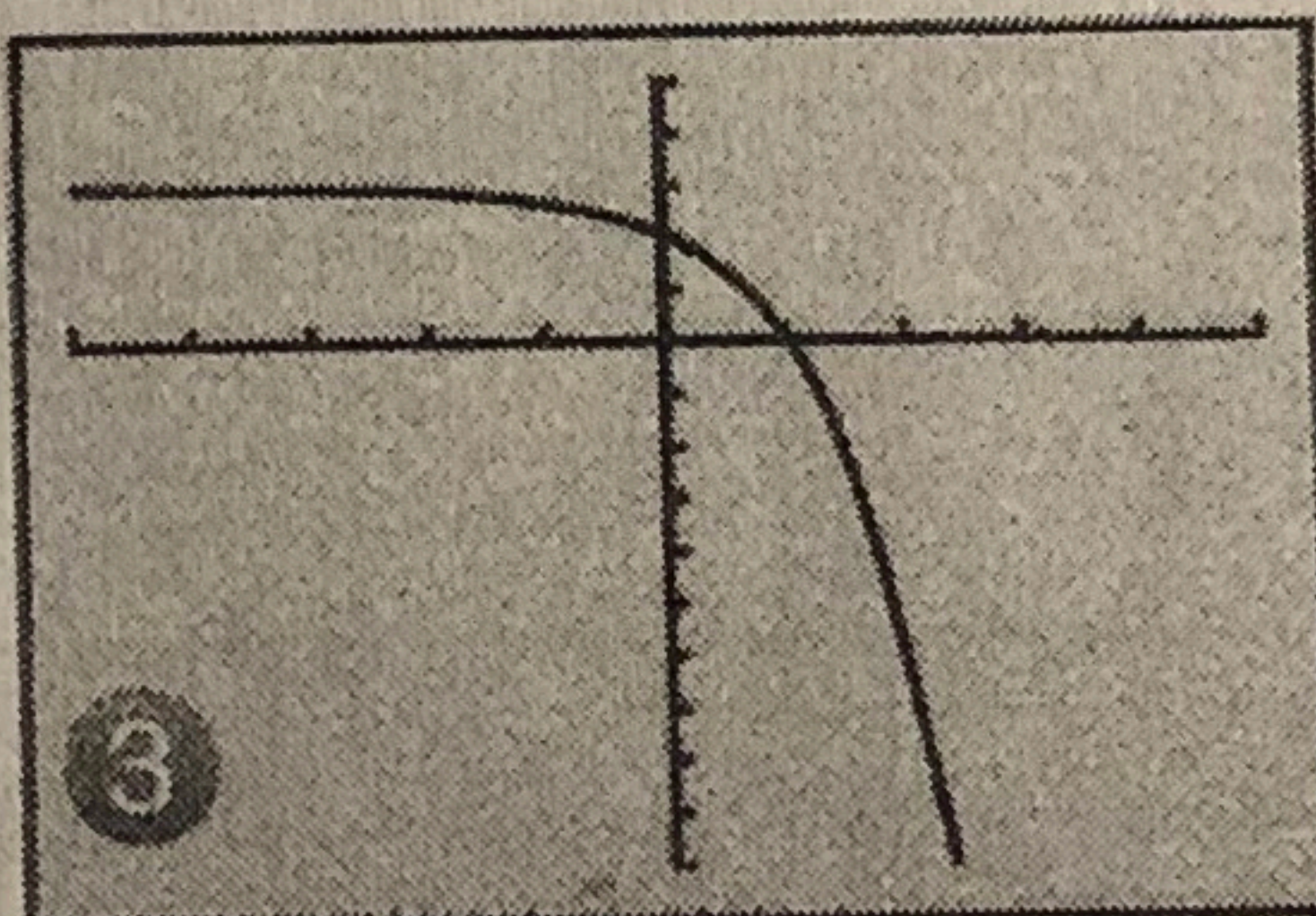
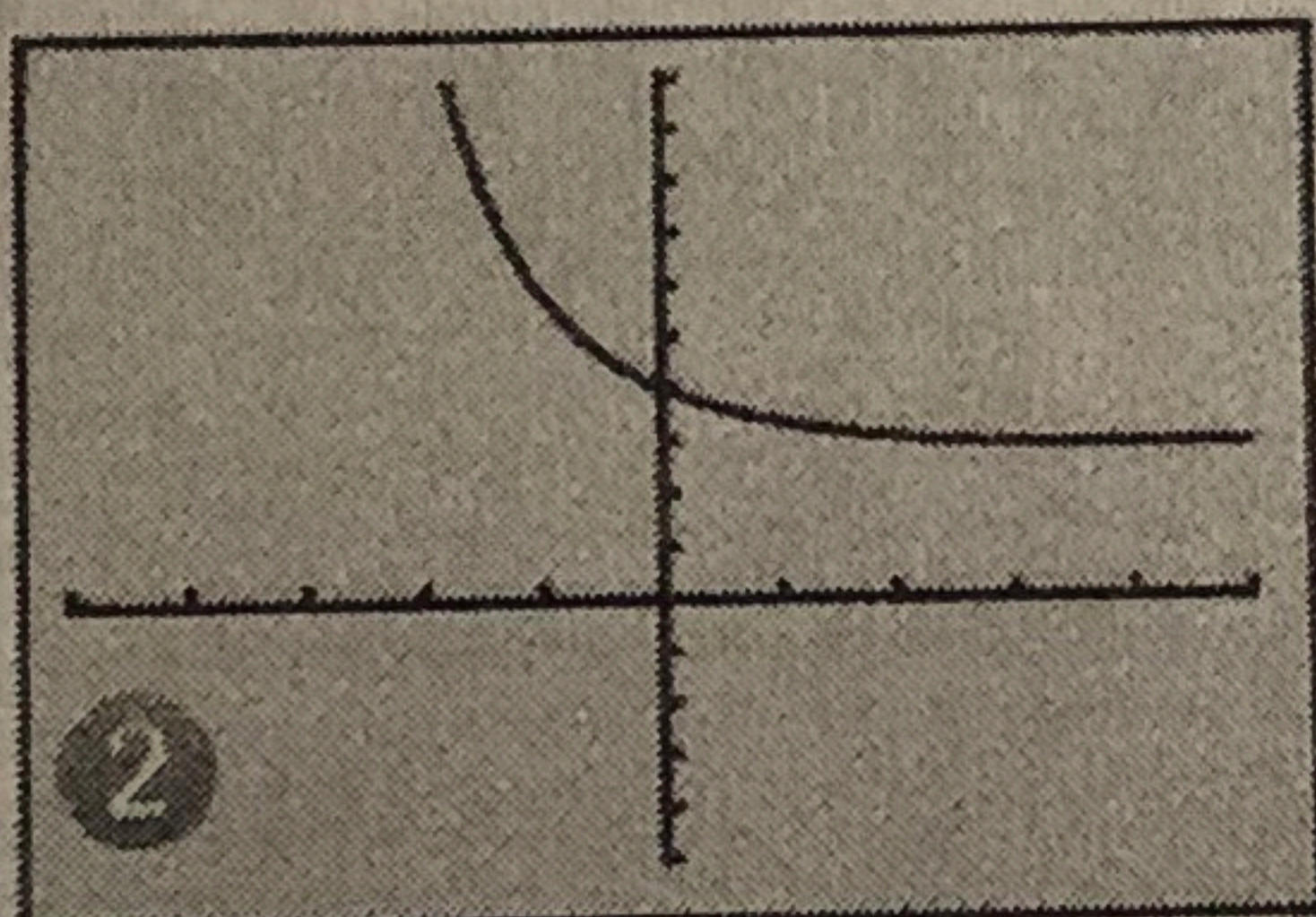
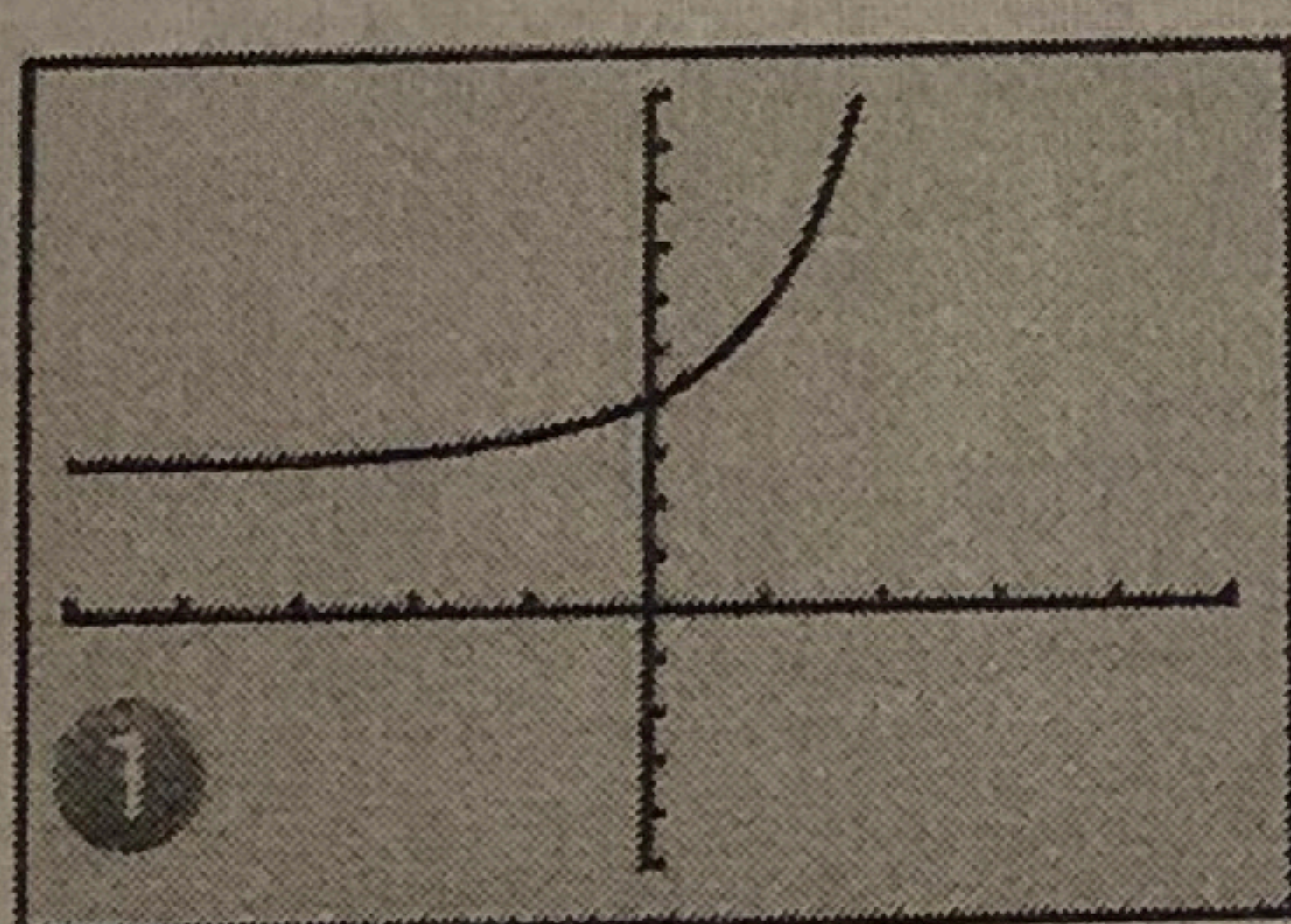
b)  $f(x) = -2(2)^x$



2) Below are the graphs of 4 exponential functions whose rules are in the form

(4 pts)

$f(x) = ac^{b(x-h)} + k$ .



For each case, determine the signs of parameters a and b when  $0 < c < 1$  and  $c > 1$

	Graph 1	Graph 2	Graph 3	Graph 4
when $0 < c < 1$	a = + b = -	a = + b = +	a = - b = -	a = - b = +
when $c > 1$	a = + b = +	a = + b = -	a = - b = +	a = - b = -

3) Points (0.5, 3.2) and (-1, 0.05) belong to an exponential function in the form of  $y = ac^x$ .

(2 pts)

Find the rule.

$$\begin{aligned} 3.2 &= ac^{0.5} \\ 0.05 &= ac^{-1} \\ \hline 64 &= c^{1.5} \\ c &= 16 \end{aligned}$$

$$\begin{aligned} 3.2 &= a(16)^{0.5} \\ a &= 0.8 \end{aligned}$$

$$\therefore f(x) = 0.8(16)^x$$

4) For each of the three functions below, state the range; whether the function is increasing or decreasing; whether it has a zero (not the value); the y-intercept & the equation of the asymptote. (4 pts)

	Range	Increasing Decreasing?	Zero? (Y or N)	Y-intercept	Asymptote
$f(x) = 3(2)^x + 14$	$]14, \infty[$	Increasing	N	$(0, 17)$	$y = 14$
$g(x) = -(3)^{2x+8}$	$] -\infty, 0[$	Decreasing	N	$(0, -6561)$	$y = 0$
$h(x) = -\left(\frac{1}{2}\right)^{2x} + 1$	$] -\infty, 1[$	Increasing	Y	$(0, 0)$	$y = 1$

5) Solve the following exponential functions (16 pts)

a)  $5^{-2(x+3)} = 625$

$5^{-2(x+3)} = 5^4$

$-2(x+3) = 4$

$x+3 = -2$

$x = -5$

c)  $(\sqrt{4})^{x+1} - 64 = 0$

$4^{1/2(x+1)} = 64$

$4^{1/2(x+1)} = 4^3$

$1/2(x+1) = 3$

$x+1 = 6$

$x = 5$

e)  $4^{x^2-9} = 1$

$4^{x^2-9} = 4^0$

$x^2-9 = 0$

$(x+3)(x-3) = 0$

$x = -3, x = 3$

g)  $0.25(5)^x > 31.25$

$5^x = 125$

$5^x = 5^3$

$x = 3$

Test  $x=0$

$0.25(5)^0 > 31.25$

$0.25 > 31.25$

False

$\therefore ]3, \infty[$

b)  $-\left(\frac{1}{3}\right)^x = -(3)^7$

$\left(\frac{1}{3}\right)^x = (3)^7$

$3^{-x} = 3^7$

$-x = 7$

$x = -7$

d)  $2^{x-2} = 32^{x+2}$

$2^{x-2} = 2^{5(x+2)}$

$x-2 = 5x+10$

$-12 = 4x$

$-3 = x$

f)  $2 \times 2^{x^2-5} = 8^x$

$2^{x^2-5+1} = 2^{3x}$

$x^2-4 = 3x$

$x^2-3x-4 = 0$

$(x-4)(x+1) = 0$

$x = 4, x = -1$

h)  $2\left(\frac{1}{3}\right)^{x+2} - 18 < 0$

$\left(\frac{1}{3}\right)^{x+2} = 9$

$3^{-(x+2)} = 3^2$

$-(x+2) = 2$

$x = -4$

Test  $x=0$

$2\left(\frac{1}{3}\right)^{0+2} - 18 < 0$

$-17.7 < 0$

$] -4, \infty[$

6) A fur dealer tells a customer that since her store opened 15 years ago, the average price of a lynx skin has risen. Over the past 15 years, the price of a lynx skin has increased according to the rule  $p = 30(1.25)^t$ , where  $t$  is the time in years. What was the price of a lynx skin 10 years after the store opened? (4 pts)

$$p = 30(1.25)^{10}$$

$$p = 30(9.313225746)$$

$$p = 279.40$$

∴ The price was \$279.40

7) Given  $f(x) = 3(0.5)^{x-2} + 10$  and  $g(x) = -3x - 5$  solve for  $f \circ g(x) = 13$ . (4 pts)

$$f \circ g(x) = 13$$

$$3(0.5)^{(-3x-5)-2} + 10 = 13$$

$$3(0.5)^{-3x-7} = 3$$

$$(0.5)^{-3x-7} = 1$$

$$(0.5)^{-3x-7} = (0.5)^0$$

$$-3x - 7 = 0$$

$$-3x = 7$$

$$x = -7/3$$

8) The temperature of a cup of tea decreases exponentially as time passes. The relation between the time  $x$  in minutes & temperature  $T$  is defined by:

$$T = (\text{Tea's original Temp} - \text{Room Temp}) \times 0.9^x + \text{Room Temp}$$

At precisely 6:00pm, one cup of tea is placed on a table & another is placed in a refrigerator. Both cups of tea are  $100^\circ\text{C}$ . The table is in a room whose temperature is  $20^\circ\text{C}$ . The refrigerator is at a temperature of  $5^\circ\text{C}$ .

At 10 minutes, what is the difference in temperature between the two cups of tea? (4 pts)

Table

$$T = (100 - 20) \times 0.9^{10} + 20$$

$$T = 47.89^\circ$$

Fridge

$$T = (100 - 5) \times 0.9^{10} + 5$$

$$T = 38.12^\circ$$

$$47.89 - 38.12 = 9.77^\circ$$

∴ The difference is  $9.77^\circ$

9) In 1960 Adrienne bought a new convertible for \$8000. Over the next 6 years, her car depreciated each year by 15% of the previous year's value. During the next 4 years, the value of the car remained the same. Then because it was so rare & beautiful, her car was classified as a collector's item. From this point on, the value of her shiny old convertible increased by 10% each year.

What was the value of the car 30 years after it was purchased?

(4 pts)

$$\begin{aligned} \text{After 6 yrs} \\ P &= 8000 \left(1 - \frac{15}{100}\right)^6 \\ P &= \underline{\$3017.20} \end{aligned}$$

$$\begin{aligned} \text{After 10 yrs} \\ P &= \$3017.20 \end{aligned}$$

$$\begin{aligned} \text{After 30 yrs} \\ P &= 3017.20 \left(1 + \frac{10}{100}\right)^{20} \\ P &= \$20298.21 \end{aligned}$$

$\therefore$  The Value is \$20298.21

10) Given  $f(x) = ac^{x-h} + k$  where  $c > 1$  and  $a < 0$ ,  $b < 0$ ,  $k > 0$  and  $h > 0$

(4 pts)

State whether the function is increasing or decreasing.

Increasing

State the range.

$]-\infty, k[$

State the domain.

$]-\infty, \infty[$

