Logarithmic Functions Practice Questions

Log Basics

1) Re-write each of the following equalities in logarithmic form.

a) $3^4 = 81$

$\log_3 81 = 4$

b) $s^v = w$

$\log_s w = v$

c) $5^{3/2} = \sqrt{125}$

$\log_5 \sqrt{125} = \frac{3}{2}$

d) $\left(\frac{1}{3}\right)^3 = \frac{1}{27}$

$\log_{1/3} \frac{1}{27} = 3$

e) $3^0 = 1$

$\log_3 1 = 0$

f) $\left(\frac{1}{4}\right)^{-4} = 256$

$\log_{1/4} 256 = -4$

2) Re-write each of the following equalities in exponential form

a) $\log_2 32 = 5$

$2^5 = 32$

b) $\log 1000 = 3$

$10^3 = 1000$

c) $\log_4 \frac{1}{4} = -1$

$4^{-1} = \frac{1}{4}$

d) $\log_5 1 = 0$

$5^0 = 1$

e) $\log_2 \frac{1}{16} = -4$

$2^{-4} = \frac{1}{16}$

f) $\log_3 3^4 = 4$

$3^4 = 3^4$

3) Use your calculator to evaluate each logarithm to 4 decimal places.

a) $\log 5$

$= 0.6990$

b) $\ln 3$

$= 1.0986$

c) $\log_3 90$

$= 4.0960$

d) $\log_3 \frac{1}{27}$

$= -3.000$

e) $\ln e$

$= 1.000$

f) $\log_{\frac{1}{3}} 9$

$= -2.000$

4) Solve each equation for x

a) $\log_{\frac{1}{3}} x = 4$

$x = \frac{1}{81}$

b) $\log_x 64 = 3$

$x = 4$

c) $\log \sqrt{8} = x$

$x = 0.4515$

d) $3^x = 12$

$x = 2.2619$

e) $x^5 = 0.5$

$x = 0.8706$

f) $\ln x = 10$

$x = 22 - 26.4658$

g) $\log_3 -7 = x$

no solution

h) $\log_3 7 = x$

$x = 1.7712$

i) $\log x^2 = 10\,000$

$x = 10^{5000}$

5) Re-write each of the following expressions using a single logarithm

a) $4 \log_3 6$ $= \log_3 1296$	b) $\log 5 + \log 5 + \log 5$ $= \log 125$	c) $4 \log_3 81 - \log_3 9$ $= \log_3 9^7$
d) $4 \log_m x - 2 \log_m x$ $= m\,x^2$	e) $2 \log a + 5 \log b$ $= \log a^2 b^5$	f) $3 \log x + \frac{1}{2} \log y$ $= \log x^3 y^{1/2}$
g) $2 \log_a m + \log_a n - 5 \log_a p$ $= \log_a \frac{m^2 n}{p^5}$	h) $\log(x^2 - 1) - \log(x - 1)$ $= \log(x + 1)$	i) $\frac{\log_a 16}{\log_a 4} - \log_4 2$ $= \log_4 8$
j) $\log_4 x + \log_6 20$ $= \log_4 x + 1.6720$		k) $\log_2 x + \log_4 y - \log_{16} z$ $= \log_2 \frac{x y^{1/2}}{z^{1/4}}$

6) For each of the following cases, indicate whether the function is increasing or decreasing.

a) $f(x) = \log_2 x$
Increasing

b) $g(x) = \log_{0.3} x$
Decreasing

c) $h(x) = 2\ln x + 9$
Increasing

d) $i(x) = 3\log_{0.5}(4 - x) + 1$
Decreasing

e) $j(x) = -\ln(x - 7)$
Decreasing

f) $k(x) = \log(x + 5) + 8$
Increasing

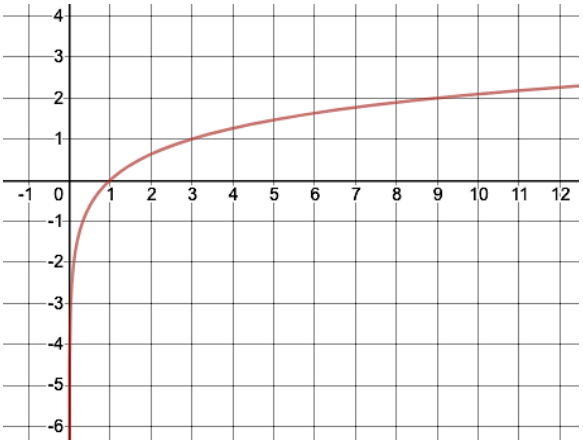
7) Find the equation of the asymptote of the curve associated with each of the functions defined below.

a) $f(x) = -\log\big(4(x + 3)\big) + 5$
$x = -3$

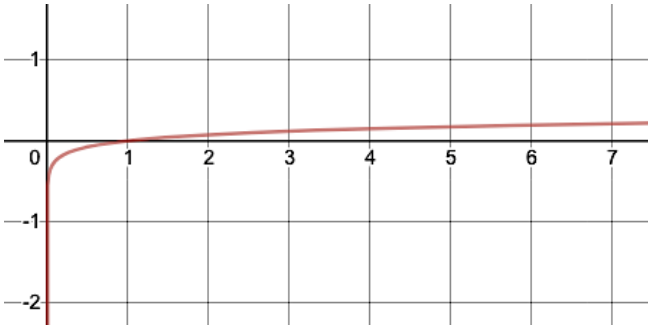
b) $g(x) = 5\log(2x + 8) - 6$
$x = -4$

8) Sketch the graphs of the following functions.

a) $\log_3 x$

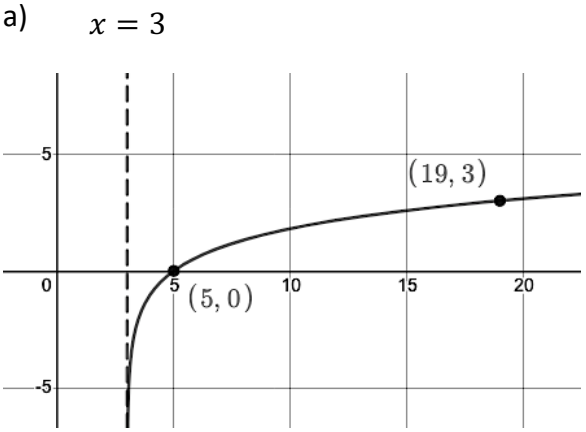


b) $\log x^{1/4}$



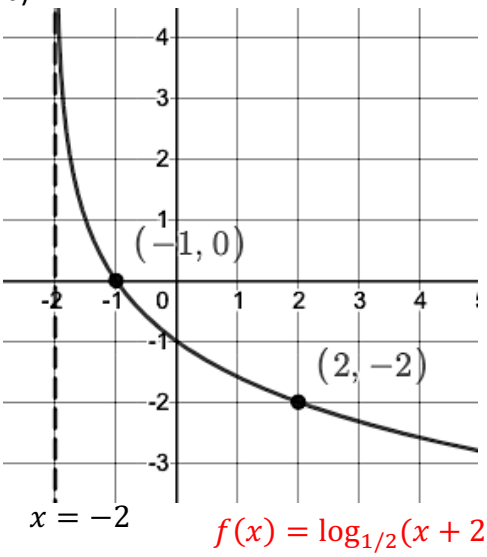
9) Determine the rule of each of the logarithmic functions represented in the graphs below.

a)



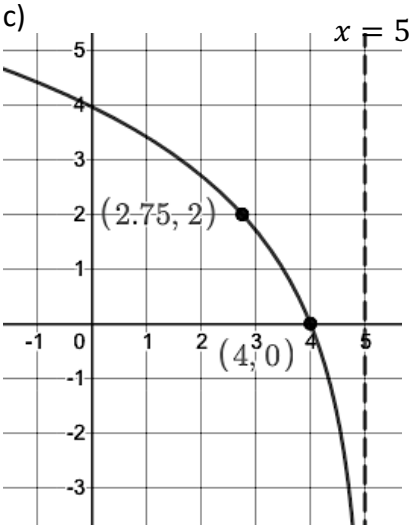
$f(x) = \log_2 0.5(x - 3)$

b)



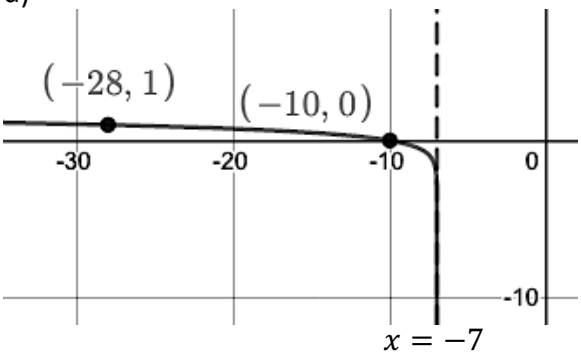
$f(x) = \log_{1/2}(x + 2)$

c)



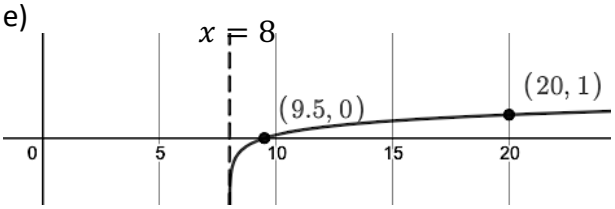
$f(x) = \log_{1.5} -(x - 5)$

d)



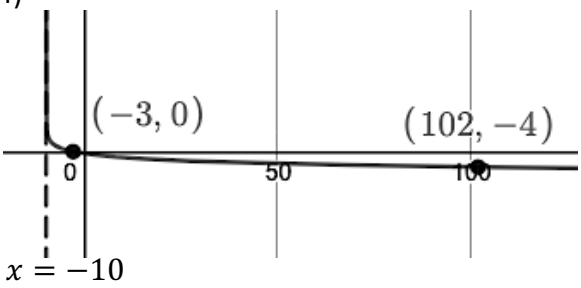
$f(x) = \log_7 -\frac{1}{3}(x + 7)$

e)



$f(x) = \log_8 \frac{2}{3}(x - 8)$

f)



$f(x) = \log_{1/2} \frac{1}{7}(x + 10)$

10) Solve the following:

a) Given $f(x) = \log_{\frac{1}{2}}(2(x+4)) - 10$

Determine when $f(x) = 14$

$x = -3.999$

(or no solution, depending on rounding)

b) Given $g(x) = -2\log_3\left(-\frac{1}{2}(x+2)\right) + 4$

Determine when $g(x) = -6$

$x = -488$

c) Given $h(x) = -\frac{1}{2}\log_{0.7}(3x+2) - 10$

Determine when $h(x) = -5.8$

$x = 6$

d) Given $i(x) = \log(2x+3)$

and $j(x) = \log 4 - x$

Determine when $i(x) = j(x)$

$x = 0.333$

11) Solve the following inequalities:

a) $20\log(x-5)+30 \geq 50$

$[15, \infty[$

b) Determine the interval over which $-2\log_{0.3}(3x+4)-8 \leq 2$

$\left]-\frac{4}{3}, 135.8409\right]$

c) $-0.5\ln 2x < -6$

$]81377.3957, \infty[$

d) $-3\log x \leq 21$

$[0.00000001, \infty[$

c) Given $f(x) = -2\log_2(-x)$ and $g(x) = 2\log_4(x+6)-2$

Determine the interval over which $f(x) \leq g(x)$.

$[-5.616, -0.354]$

12) Solve the following exponential functions:

a) $\left(\frac{1}{2}\right)^{x+2} = 28$

$x = -6.8074$

b) $13^{\left(\frac{2-x}{4}\right)} = \frac{3}{8}$

$x = 3.5296$

c) $21^{4x} = 0.35$

$x = -0.0862$

d) $3\left(\frac{1}{2}\right)^{-2x+5} = 21$

$x = 3.9037$

e) $3^{x+5} = 4^{x+3}$

$x = 4.636$

f) $2.5(10)^{x} = 6(e^{4x-3})$

$x = -1.6311$

13) The value, V (in $) of an investment changes according to the rule $V = 15\ 000\ (1.015)^{2t}$ where t is time (in years). When is the value of the investment:

a) $15 000 b) $20 000 c) $22,000

a) $t = 0\ years$

b) $t = 9.6611\ years$

c) $t = 12.8619\ years$

14) The town of Springfield is growing at a rate of 6.5% per year. How many people are there in Springfield now, if there will be 15 000 people in 4.5 years?

There are 11 298 people

15) A delivery company estimates that its road equipment depreciates by 30% each year. After 5 years one of its transport trucks was worth $12 605.
a) How much did the company pay for this truck?
b) After how much time was the truck worth 10% of its original cost?

a) $74 998.51

b) 6.46 years

16) To produce maple syrup, 1000L of sap is boiled until 97.5% of its initial quantity has evaporated. With each hour of boiling, the quantity of sap decreases by 10% in relation to the amount from the previous hour. When should the boiling process be stopped?

The boiling process should be stopped after 35.01 hours.

17) State the rule for the inverse of each of the functions defined below.

a) $f(x) = \log_4 x$

$f^{-1}(x) = 4^x$

b) $g(x) = \log x^{1/3}$

$g^{-1}(x) = 10^{3x}$

c) $h(x) = \log_6 x$

$h^{-1}(x) = 6^x$

d) $f(x) = 2\log_5 3(x-1)$

$f^{-1}(x) = \frac{1}{3}(5^{x/2}) + 1$

e) $g(x) = -\log 2x + 15$

$g^{-1}(x) = \frac{1}{2}(10^{-(x-15)})$

f) $h(x) = -\ln -(x+3) - 10$

$h^{-1}(x) = -(e^{-(x+10)}) + 3$

18) Camilla deposits a $1500 scholarship she received into an account with an annual interest rate of 3.5% compounded every 6 months. When does the value of the investment reach $2500?

After 14.7229 years