## Check Your Understanding

## Graph Theory - Basics, Euler paths and circuits, Hamiltonian paths and circuits

The table below identifies some key concepts from this unit. Complete each question, check your answers, and get help as needed.

| Key Concepts | Basic <br> Questions | Intermediate <br> Questions | Advanced <br> Questions |
| :---: | :---: | :---: | :---: |
| Paths | 1 a | 1 b | 1 c |
| Circuits | 2 a | 2 b | 2 c |
| Euler Paths | 3 a | 3 b | $3 \mathrm{c}, 3 \mathrm{~d}$ |
| Euler Circuits | 4 a | 4 b | $4 \mathrm{c}, 4 \mathrm{~d}$ |
| Hamiltonian Paths | 5 a | 5 b | $5 \mathrm{c}, 5 \mathrm{~d}$ |
| Hamiltonian Circuits | 6 a | 6 b | $6 \mathrm{c}, 6 \mathrm{~d}$ |

1a. Find a path from A to F.


1b. There are 5 rooms (A, B, C, D, E) connected by a series of hallways.

- Room A is connected to rooms B, D
- Room B is connected to rooms A, C, D, E
- Room C is connected to rooms B, E
- Room D is connected to rooms A, B, E
- Room E is connected to rooms B, C, D

Find a path to get from room C to room A .

1c. Find a simple path from $C$ to $E$ that contains 5 edges.


2a. Find a circuit using at least 3 edges.
2b. Find a simple circuit starting and ending at D that uses at least 6 edges.


2c. Six towns (A, B, C, D, E, F) are connected by a series of highways.

- A is connected to D, E
- $B$ is connected to $D, F$
- $C$ is connected to $D, F$
- $D$ is connected to $A, B, C, E$
- E is connected to $\mathrm{A}, \mathrm{D}$
- F is connected to $\mathrm{B}, \mathrm{C}$

Find a circuit that starts and ends at A and visits each town at least once.

3a. Find an Euler path in the graph below.


3b. Find an Euler path in the graph below.


3c. An Euler path does not exist in the graph below. What edge(s) would need to be added in order for an Euler path to exist?


4a. Find an Euler circuit in the graph below. 4b. Find an Euler circuit in the graph below.


4c. An Euler circuit does not exist in the graph below. What edge(s) would need to be added in order for an Euler circuit to exist?


5a. Find a Hamiltonian path in the graph below.


5b. Find a Hamiltonian path in the graph below.


5c. A Hamiltonian path does not exist in the graph below. What edge(s) would need to be added in order for a Hamiltonian circuit to exist?


6a. Find a Hamiltonian circuit in the graph below.


6b. Find a Hamiltonian circuit in the graph below.


6c. A Hamiltonian circuit does not exist in the graph below. What edge(s) would need to be added in order for a Hamiltonian circuit to exist?


## Answer Key

1a) ADEF (and other options)

1b) CBA (and other options)
1c) CFABDE
2a) ADBA (and other options)
2b) DECDFABD (and other options)
2c) ADCFBDEA (and other options)
3a) BDEADCF (and other options)
3b) EADEBFCDBCAB (and other options)
3c) GD (and other options)
4a) ACFAEBDA (and other options)
4b) GDBEDCFACEAG (and other options)
4c) $A D$ (other options require multiple lines to be added)

5a) AFBEDC (and other options)
5b) BADCFJHIGE (and other options)
5c) DF (and other options)
6a) DCFABED (and other options)
6b) ADCJFEIGHBA (and other options)

6c) Add AD, DC, JF, and FE (and other options)

