## Check Your Understanding

## Graph Theory - Critical Path

The table below identifies some key concepts from this unit. Complete each question, check your answers, and get help as needed.

| Key Concepts | Basic <br> Questions | Intermediate <br> Questions | Advanced <br> Questions |
| :---: | :---: | :---: | :---: |
| Find length of paths | 1 |  |  |
| Find minimum time necessary | 2 |  |  |
| Create a graph | 3 | 4,5 | 6,7 |
| Find critical path |  |  |  |

1) Find the length of every path starting at $A$ and ending at $K$.

2) The graph below represents the steps necessary for a student to complete her final math project, with the weights of each edge given in days (with A being the first step and J the last).
Some steps can be completed at the same time as other steps. What is the minimum amount of time necessary for the student to complete all the steps in the project?

3) Each morning, Xavier follows the same routine to get ready for school, shown in the table below.

| Step | Description | Time (min.) | Prior Steps |
| :--- | :--- | :--- | :--- |
| A | Wake up | 2 | None |
| B | Make coffee | 3 | A |
| C | Make breakfast | 5 | A |
| D | Shower | 10 | B and C |
| E | Decide on outfit | 3 | D |
| F | Brush teeth | 2 | D |
| G | Get dressed | 5 | E and F |
| H | Ready to go! | None | G |

Create a graph to represent Xavier's morning routine.
4) There are several main steps in designing a new car, listed in the table below.

| Step | Description | Time (months) | Prior Steps |
| :--- | :--- | :--- | :--- |
| A | Draw and design | 3 | None |
| B | Digital design model | 4 | A |
| C | Clay model | 2 | A |
| D | Interior model | 1 | B |
| E | Lights and glass design | 1 | D |
| F | Color and trim design | 2 | C |
| G | Show car development | 5 | E and F |
| H | Car development done | none | G |

What is the minimum amount of time needed to develop a new car?
5) Sierra is preparing a dinner for her friends. The following table shows the different steps involved in preparing and serving the dinner, along with the order in which the steps must be completed.

| Tasks | Time (min.) | Prerequisite Steps |
| :--- | :--- | :--- |
| A. Mix brownies | 15 | none |
| B. Bake brownies | 45 | A |
| C. Chop vegetables | 10 | none |
| D. Season vegetables | 5 | C |
| E. Prepare chicken | 15 | none |
| F. Cook chicken | 90 | E |
| G. Peel potatoes | 10 | none |
| H. Boil potatoes | 15 | G |
| I. Mash potatoes | 5 | H |
| J. Remove chicken from <br> oven and let rest | 10 | F |
| K. Roast vegetables | 15 | D, J |
| L. Serve meal | None | B, I, K |

What is the minimum amount of time Sierra needs to prepare and serve the dinner?
6) Izzy owns a café and every morning there are certain tasks that must be completed in order to open for the day.

| Task | Time (min) | Prior Steps |
| :--- | :--- | :--- |
| A | 10 | None |
| B | 8 | A |
| C | 7 | B |
| D | 5 | B |
| E | 8 | B |
| F | 4 | E |
| G | 6 | C and D |
| H | 3 | F and G |
| I | 5 | H |
| J | 3 | I |

Izzy can hire one person to shorten the total time required to open the café. Izzy has two options:
Option 1: Hire someone to help with task $C$ and save 4 minutes on that task
Option 2: Hire someone to help with tasks E and F, and save 3 minutes and 1 minute, respectively.

Which option should Izzy choose and by how many minutes will Izzy's minimum time diminish?
7) Aidan is the chief electrical engineer working on the construction of a new apartment building. The table below shows the different steps involved in the electrical work for the building, as well as the time necessary for each step and the required prior step(s). Several steps can be carried out at the same time.

| Step | Time (days) | Prior Step(s) |
| :--- | :--- | :--- |
| A | 12 | None |
| B | 17 | None |
| C | 50 | None |
| D | 20 | A |
| E | 19 | B |
| F | 18 | D and E |
| G | 15 | F |
| H | 15 | C |
| I | 13 | G |
| J | 30 | G |
| K | 14 | H |
| L | 21 | I, J, K, |

Aidan decides to hire an additional employee to help with Step B. Now the time for Step B is reduced by 6 days.

How much time is saved when Aidan hires the additional employee?

## Answer Key

1) ABDEHIJK: 29; ABDFGJK: 30; ACDEHIJK: 26; ACDFGJK: 27
2) Minimum time necessary is 13 days
3) 


4) 14 months
5) 130 minutes (or 2 hours 10 minutes)
6) Option 1 saves 1 minute
7) 4 days

