Check Your Understanding Probability – Basic, And, Or, and Conditional Probability

Key Concepts	Basic Questions	Intermediate Questions	Advanced Questions
Probabilities with "And"	1, 2, 5	3, 4	10, 11
Probabilities with "Or"	1, 2, 5	3, 4	11
Venn Diagrams	2	3	4
Contingency Tables	5	6	7
Conditional Probabilities		6, 7, 8	
Expected Gain and Fairness	9	10, 11	12

1) Given
$$P(A) = \frac{2}{7}$$
 and $P(B) = \frac{3}{17}$ determine:

a. $P(A \cap B)$

b. $P(A \cup B)$

2) Counsellors are hired for a large summer camp; 25 students applied for the positions.To be hired, they must be qualified in one, two or three areas of expertise:

- Teaching art
- Coaching sports
- Administering first aid

The diagram shows the number of students with each expertise.

Which of the following statements is true?



- A) The number of students that have expertise in teaching art and coaching sports is 1.
- B) The number of students that have expertise in coaching sports or administering first aid is 17.
- C) The number of students that have expertise in teaching art, administering first aid and coaching sports is 8.
- D) The number of students that have no expertise in these areas is 4.

3) A school offers three sports for the Sec V students to play: soccer, baseball and hockey. There are 100 Sec V students. A student can choose to play more than one sport. Here is a list of how many of those students signed up for the sports options:

42

46

45

20

- Soccer
- Baseball
- Hockey
- All 3 sports
- Soccer and baseball 30
- Soccer and hockey 24
- Baseball and hockey 29



- a) How many students signed up only for hockey?
- b) How many students did not sign up for any sports?
- 4) 55 people were asked which colors they like: green, orange, and blue.
 - 4 people like all three colors
 - 7 people like green and orange
 - 11 people like blue and orange
 - 25 people like orange
 - 22 people like green
 - 25 people like blue
 - 37 people like green or blue

Create a Venn diagram and determine:

- a) $P(Green \cap Blue)$
- **b)** $P(Blue \cup Orange)$

5) In a carnival game, players were asked to flip a coin and pick a marble (from a bag containing green, blue, and red marbles). 30 people played and the results are given in the table below.

	Green	Blue	Red	Total
Heads	3	2	7	12
Tails	1	6	11	18
Total	4	8	18	30

Determine:

- a) $P(Heads \cap Blue)$
- **b)** $P(Green \cup Tails)$
- 6) A survey was conducted at a nearby university to learn about the commuting habits of both students and employees. The respondents were asked whether they use public transit or drive a car. Given the respondent is a student, there is a probability of $\frac{4}{7}$ that he or she uses public transport.

The incomplete table below illustrates additional findings of the survey.

	PUBLIC TRANSIT	CAR	ΤΟΤΑΙ
STUDENT			3934
EMPLOYEE			1686
TOTAL	2856	2764	5620

- a) How many students use public transit?
- b) What is the probability that the respondent is a student, given that the respondent drives a car?

- A school is offering 3 different trip destinations: Paris, Los Angeles and New York City. Students may only choose only 1 destination. The teachers have collected the following data based on destination and grade.
- There are 750 students who signed up.
- Twice as many grade 10 students as grade 11 students signed up.
- The probability that a student signed up for Paris is $\frac{2}{5}$.
- The probability that a student signed up for Los Angeles is the same as the probability that they signed up for New York City.
- Given that a student chose Paris, the probability that the student is in Grade 11 is 30%.
- Given that a student is in Grade 10, the probability that they signed up for Los Angeles is 0.25.

Create a contingency table.

Given that a student chosen at random is in Grade 11, what is the probability that they signed up for New York City?

8) Given the dependent events, A and B, where $P(A) = \frac{7}{8}$, $P(B) = \frac{3}{11}$, and $P(A \cap B) = \frac{15}{75}$ Determine:

a) P(A|B)

b) **P**(**B**|**A**)

9) You are trying to determine whether to play a carnival game or not. You must pay \$10 to play the game. If you win, you will get a prize worth \$65. If you lose, you get nothing.

$$P(win) = \frac{1}{6}$$
 and $P(lose) = \frac{5}{6}$

What is the expected gain of this game? Is it a fair game?

10) You are analyzing a game where you pick a card from a standard deck of cards and you spin a spinner (divided into 4 equal sections, labeled A, B, C, D). The cost to play the game is \$5. If you win, you get \$20.

To win, you must pick a diamond card and the spinner must land on C.

Determine: a) P(win)

b) *P*(*lose*)

c) Expected Gain

d) Does this game favor the player or the dealer?

11) You are analyzing a game where you pick a marble (from a bag containing 3 blue marbles, 5 red marbles, and 8 yellow marbles) and you roll two dice.

The game costs \$4 to play.

You win \$6 if you pick a blue marble or roll a sum of 7.

You win \$20 if you pick a red marble and roll a 12.

Is this a fair game? Explain.

12) While at a carnival, Serena finds two games of chance that will allow her to win a prize. She decided to play either The Colour Is Right or The Fruit Wheel.

The amount of money Serena must bet to play either game is the same.

THE COLOUR IS RIGHT

After placing a bet, the player randomly selects a marble from a bag.

The bag contains 5 red marbles, 2 blue marbles and 3 yellow marbles.

- If a red marble is chosen, the player does not receive any money.
- If a blue marble is chosen, the player receives \$28.
- If a yellow marble is chosen, the player receives the value of the bet.

This game is fair.

THE FRUIT WHEEL

This game consists of spinning a wheel after placing a bet. The wheel is divided into 12 congruent sectors.

- If the pointer stops on an apple, the player receives \$4.
- If the pointer stops on a strawberry, the player does not receive any money.
- If the pointer stops on a cherry, the player receives \$12.
- If the pointer stops the JACKPOT section, the player receives \$40.



Is The Fruit Wheel game to Serena's advantage, to Serena's disadvantage, or is it fair? Justify your reasoning.

Answer Key

1)

a. $P(A \cap B) = \frac{6}{119}$ b. $P(A \cup B) = \frac{7}{17}$

- 2) B
- 3)
- a. 12 students for hockey only b. 30 students for none
- 4)
- a. $P(Green \cap Blue) = \frac{2}{11}$ b. $P(Blue \cup Orange) = \frac{39}{55}$
- 5)

a.
$$P(Heads \cap Blue) = \frac{1}{15}$$

b. $P(Green \cup Tails) = \frac{7}{10}$

- 6)
- a. 2248 students use public transit.
- b. $P(student|car) = \frac{843}{1382}$

7)
$$P(NYC|Gr11) = \frac{6}{25}$$

8)

a.
$$P(A|B) = \frac{11}{15}$$

b. $P(B|A) = \frac{8}{35}$

9) Expected gain is \$0.83. The game is not fair.

10)

a.
$$P(win) = \frac{1}{\frac{16}{16}}$$

- b. $P(lose) = \frac{15}{16}$ c. Expected Gain \$ 3.75
- d. The game favors the dealer.
- 11) The game is not fair as the expected gain is \$-1.88 and a fair game has an expected gain of 0.
- 12) The fruit game is to Serena's disadvantage because the expected gain is \$-0.67.