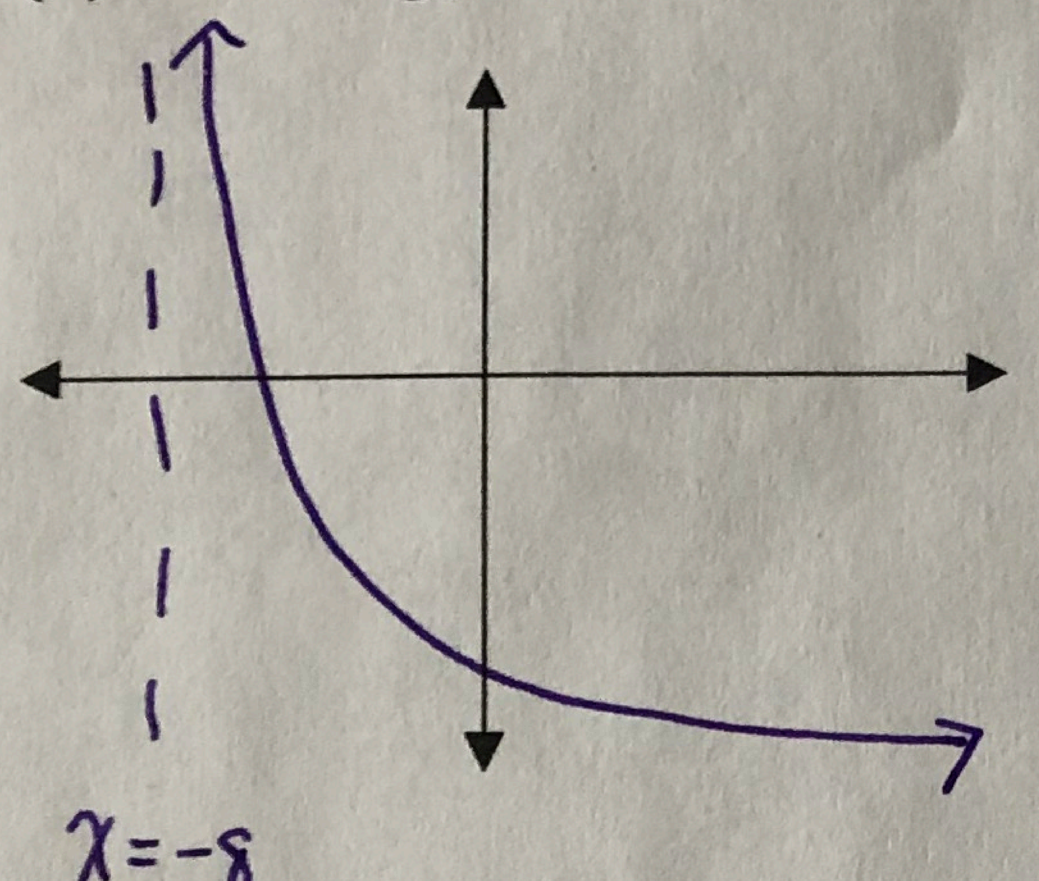
Name Key Group________ /44

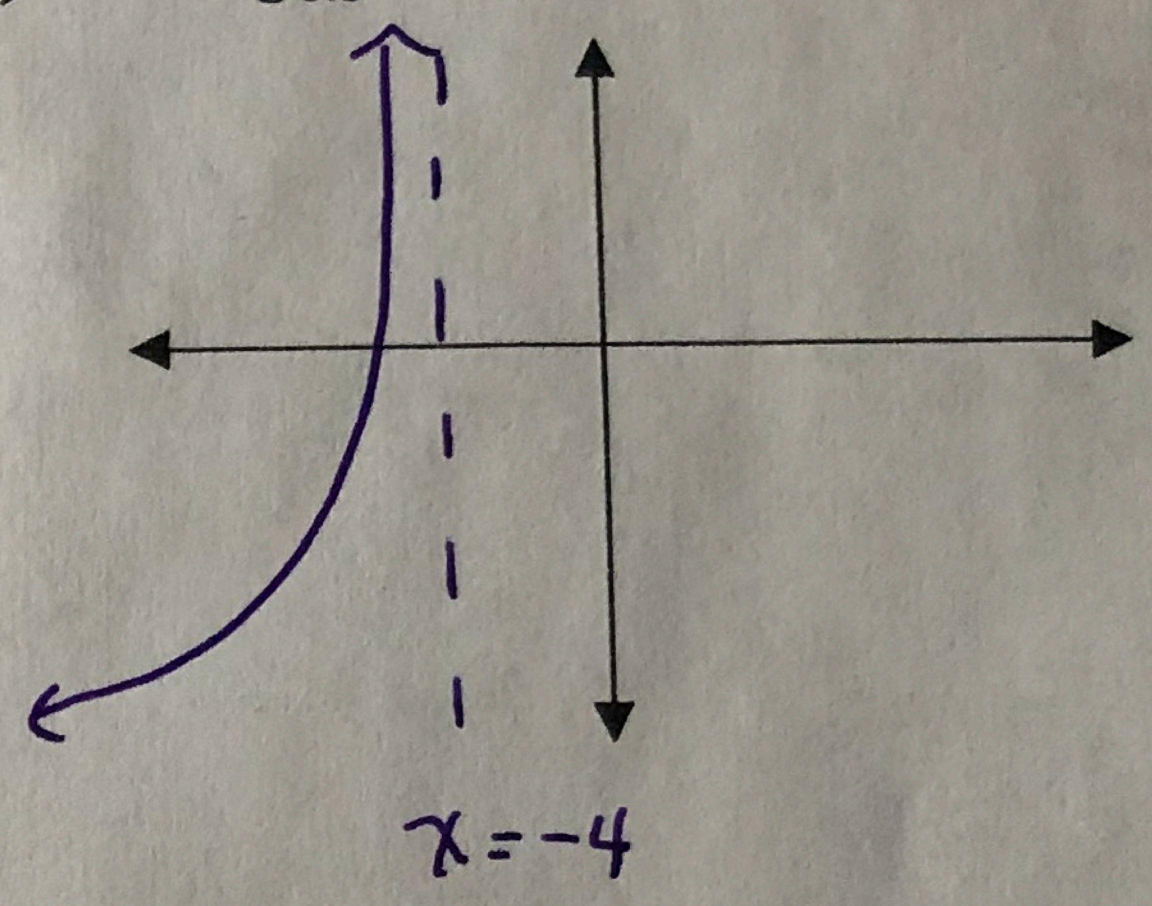
Logarithmic Functions

1) Sketch the following functions (4 pts)

a) $f(x) = -2\log_3 7(x+8) - 3$



b) $f(x) = 3\log_{0.5} -3(x+4) + 6$



2) For each of the functions below, state the domain, whether the function is increasing or decreasing, the value of the zero, whether a y-intercept exists (Y or N) and the equation of the asymptote. (4 pts)

	Domain	Increasing Decreasing	Zero	y-int?	Asymptote
$f(x) = 3\log_3(x-1)$	$]1, \infty[$	Increasing	$x=2$	N	$x=1$
$g(x) = -2\log_4(2x+4)+4$	$]-2, \infty[$	Decreasing	$x=6$	Y	$x=-2$
$h(x) = \log_{\frac{3}{4}}(-(x-3))$	$]-\infty, 3[$	Increasing	$x=2$	Y	$x=3$

3) Find the rule of a logarithmic function passing through the points $(-9, 0)$ and $(-25, 2)$, with an asymptote at $x = -7$. (4 pts)

$0 = \log_c b(-9+7)$

$0 = \log_c b(-2)$

$c^0 = -2b$

$1 = -2b$

$-\frac{1}{2} = b$

$2 = \log_c(-\frac{1}{2}(-25+7))$

$2 = \log_c(9)$

$c^2 = 9$

$c = 3$

$\therefore f(x) = \log_3(-\frac{1}{2}(x+7))$

4) Solve the following logarithmic functions (16 pts)

a) $\log_2(5x) - 4 = -2$

$\log_2(5x) = 2$

$2^2 = 5x$

$4 = 5x$

$\boxed{4/5 = x}$

Rest.

$5x > 0$

✓

b) $\log_4(x^2 + 15x) = 2$

$4^2 = x^2 + 15x$

$0 = x^2 + 15x - 16$

$0 = (x+16)(x-1)$

$\boxed{x = -16,\ x = 1}$

Rest

$x^2 + 15x > 0$

✓

c) $\log_2(x^2 + 5) - \log_2 5 = \log_2 6$

$\log_2(x^2+5) = 4.9069$

$x^2 + 5 = 30$

$x^2 - 25 = 0$

$(x+5)(x-5) = 0$

$\boxed{x = 5,\ x = -5}$

Rest

$x^2 + 5 > 0$

✓

d) $2\log_7(-3x + 9) - 2 \geq 1$

$2\log_7(-3x+9) - 2 = 1$

$\log_7(-3x+9) = 1.5$

$7^{1.5} = -3x + 9$

$18.52 = -3x + 9$

$9.52 = -3x$

$-3.17 = x$

Rest

$-3x + 9 > 0$

✓

Test let $x = 0$

$2\log_7(9) - 2 \geq 1$

$0.258 \geq 1$

X

$\boxed{]-\infty, -3.17]}$

e) $\log_6(x + 3) + \log_6(x - 2) = 1$

$\log_6(x+3)(x-2) = 1$

$6 = x^2 + x - 6$

$0 = x^2 + x - 12$

$0 = (x+4)(x-3)$

$x = -4,\ \boxed{x = 3}$

X

Rest

$x - 4 > 0$

$x + 2 > 0$

f) $\log_2(x - 3) + \log_2(2x) = 3$

$\log_2(x-3)(2x) = 3$

$8 = (x-3)(2x)$

$8 = 2x^2 - 6x$

$0 = 2x^2 - 6x - 8$

$0 = x^2 - 3x - 4$

$0 = (x-4)(x+1)$

$\boxed{x = 4}\ x = -1$

X

Rest

$x - 3 > 0$

$2x > 0$

g) $\log_2(x - 4) + \log_2(x + 2) = 4$

$\log_2(x-4)(x+2) = 4$

$16 = x^2 - 2x - 8$

$0 = x^2 - 2x - 24$

$0 = (x-6)(x+4)$

$\boxed{x = 6},\ x = -4$

X

Rest

$x - 4 > 0$

$x + 2 > 0$

h) $\log_3(x + 4) = \log_6 6 - \log_3(x + 2)$

$\log_3(x+4)(x+2) = 1$

$3 = x^2 + 6x + 8$

$0 = x^2 + 6x + 5$

$0 = (x+5)(x+1)$

$x = -5,\ \boxed{x = -1}$

X

$x + 4 > 0$

$x + 2 > 0$

6) In a manufacturing company, the assembly time (in minutes) varies throughout the training process of their employees according to the rule $t(n) = -40\log\left(\frac{n}{4}+1.5\right)+100$ where n is the number of parts assembled. How many parts must be built if the assembly time is 60 minutes?

(4 pts)

$60 = -40\log\left(\frac{n}{4}+1.5\right)+100$

$-40 = -40\log\left(\frac{n}{4}+1.5\right)$

$1 = \log\left(\frac{n}{4}+1.5\right)$

$10 = \frac{n}{4}+1.5$

$8.5 = \frac{n}{4}$

$34 = n$

Rest.

$\frac{n}{4}+1.5 > 0$ ✓

$\boxed{\therefore \text{ 34 parts are built}}$

7) In 2001, Albert invested \$4000. His investment was compounded annually and by 2009 had grown to \$5474.28. He eventually was able to triple his initial investment.

At the same time in 2001, Jocelyn invested a sum of money at the same interest rate as Albert's investment, compounded annually. In the number of years it took Albert to triple his initial investment, Jocelyn's investment grew to \$15,000.

What was Jocelyn's initial investment?

(4 pts)

Albert (rate)

$5474.28 = 4000(1+i)^8$

$1.36857 = (1+i)^8$

$1.04 = 1+i$

$i = 0.04$

rate was 4%

Albert (time to 3x)

$12000 = 4000(1.04)^x$

$3 = 1.04^x$

$\log_{1.04} 3 = x$

$x = 28$

28 years

Jocelyn

$15000 = a(1.04)^{28}$

$15000 = 2.9987a$

$a = \$5002.17$

$\boxed{\therefore \text{ Jocelyn's initial investment is } \$5002.17}$

8) Kelly loves to simplify logs! Dr. James gives her the following question for practice.

$$2\log_{\frac{1}{3}}(27)-\log_2\left(\frac{1}{8}\right)^3+\log_{27}(1)^{16}-\frac{3}{4}\log_{\frac{1}{2}}(16)$$

Kelly grumbles at the question for a little while, and then calculates a numerical answer to the above expression. What is Kelly's answer?

(4 pts)

$2\log_{1/3}(27)-\log_2(1/8)^3+\log_{27}(1)^{16}-\frac{3}{4}\log_{1/2}(16)$

$= -6 - -9 + 0 - -3$

$= -6 + 9 + 3$

$\boxed{= 6}$

9) Given $f(x) = 3(0.5^{2(x+8)}) - 4$, $g(x) = 3x + 2$, and $h(x) = f^{-1}(g(x))$
Write the rule for $h(x)$.

(4 pts)

$f^{-1}(x)$

$x = 3(0.5^{2(y+8)}) - 4$

$\frac{x+4}{3} = 0.5^{2(y+8)}$

$\log_{0.5}\left(\frac{x+4}{3}\right) = 2(y+8)$

$\frac{1}{2}\log_{0.5}\left(\frac{x+4}{3}\right) - 8 = y$

$f^{-1}(x) = \frac{1}{2}\log_{0.5}\left(\frac{1}{3}(x+4)\right) - 8$

$h(x) = \frac{1}{2}\log_{0.5}\left(\frac{1}{3}((3x+2)+4)\right) - 8$

$= \frac{1}{2}\log_{0.5}\left(\frac{1}{3}(3x+6)\right) - 8$

$= \frac{1}{2}\log_{0.5}(x+2) - 8$

$\boxed{\therefore h(x) = \frac{1}{2}\log_{0.5}(x+2) - 8}$