Grade 10
CST Math
Student Workbook

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Special thanks to M. Gervais for the use of her notes and worksheets

Review Unit – Solving Algebraic Equations

Variables are letters or symbols that represent numbers we do not know or that can change. We can use algebra to find the exact value of the variable.

Ex: Ms. James brought some oranges to school. During the day, she gave away 2 oranges. At the end of the day, Ms. James had 3 oranges. How many oranges did Ms. James bring to school?

There are two keys to solving equations:

1) To get rid of a number, you do its opposite (for example, if it was addition, use subtraction).
2) Whatever you do to one side, you do the same to the other side.

Let's look at 5 different levels.

Level 1: Variable on one side with only addition or subtraction

Get rid of the addition or subtraction by doing the opposite on both sides.

Ex:

a) $x + 2 = 3$ b) $3 + x = 7$ c) $x - 4 = 8$ d) $-7 + x = 2$

Try these questions!

1a) $x - 6 = 10$ b) $8 + x = 11$ c) $2 = -5 + x$ d) $20 = x + 3$

Review Unit – Solving Algebraic Equations

Level 2: Variable on one side with addition or subtraction and multiplication.

Get rid of the addition or subtraction first, then get rid of the multiplication by dividing.

Ex:

a) $3x + 9 = 12$ b) $4 + 2x = 10$ c) $20x - 25 = 75$ d) $-7 + 3x = 20$

Try these questions!

2a) $4x - 6 = 6$ b) $8 + 7x = 43$ c) $15 = -5 + 5x$ d) $33 = 10x + 3$

Level 3: Variables and addition or subtraction on both sides

Get rid of the variable on one side, then get rid of the addition or subtraction on the other side.

Ex:

a) $3x + 3 = 13 - 2x$ b) $4 - 4x = 18 - 6x$ c) $28x - 15 = 8x + 65$

Try these questions!

3a) $2x - 6 = 8 - 5x$ b) $8 + 7x = 3x + 28$ c) $15 - 2x = -5 + 2x$

Review Unit – Solving Algebraic Equations

Level 4: Brackets on one or both sides

Get rid of the brackets by expanding and then solve like in Level 3.

Ex:

a) $6(x-2) = 15 + 3x$ b) $3(4+3x) = 4(2+x)$ c) $2(7-8x) = 5(2x-4)$

Try these questions!

4a) $2(x+3) = 10 - 2x$ b) $7(2x+3) = 3(4x-2)$ c) $3(2-5x) = 5(4-6x)$

Level 5: Division (fractions) on one or both sides

Cross multiply then solve like Level 4.

Ex:

a) $\frac{3x+2}{4} = \frac{2-6x}{5}$ b) $\frac{4-x}{3} = \frac{5-2x}{2}$ c) $\frac{8x+3}{2} = 2 + 3x$

Try these questions!

5 a) $\frac{2+4x}{3} = \frac{3x-1}{2}$ b) $\frac{2x+7}{2} = \frac{4-6x}{4}$ c) $8x + 3 = \frac{12x-5}{5}$

Review Unit – Solving Algebraic Equations

Practice Questions

1) $x + 3 = 5$

2) $2x + 4 = x + 12$

3) $3(x + 4) = 2(5 - 2x)$

4) $\frac{2x-3}{4} = \frac{2+x}{5}$

5) $3x - 6 = 18$

6) $x + 1 = 4(2 + 4x)$

7) $\frac{18x+3}{6} = 5x - 2$

8) $4x - 3 = 13$

Review Unit – Solving Algebraic Equations
Answer Key

Questions in the Notes

1a) $x = 16$	**1b)** $x = 3$	**1c)** $x = 7$	**1d)** $x = 17$
2a) $x = 3$	**2b)** $x = 5$	**2c)** $x = 4$	**2d)** $x = 3$
3a) $x = 2$	**3b)** $x = 4$	**3c)** $x = 5$	
4a) $x = 1$	**4b)** $x = -13.5$	**4c)** $x = 0.93$	
5a) $x = 7$	**5b)** $x = -1$	**5c)** $x = -0.71$	

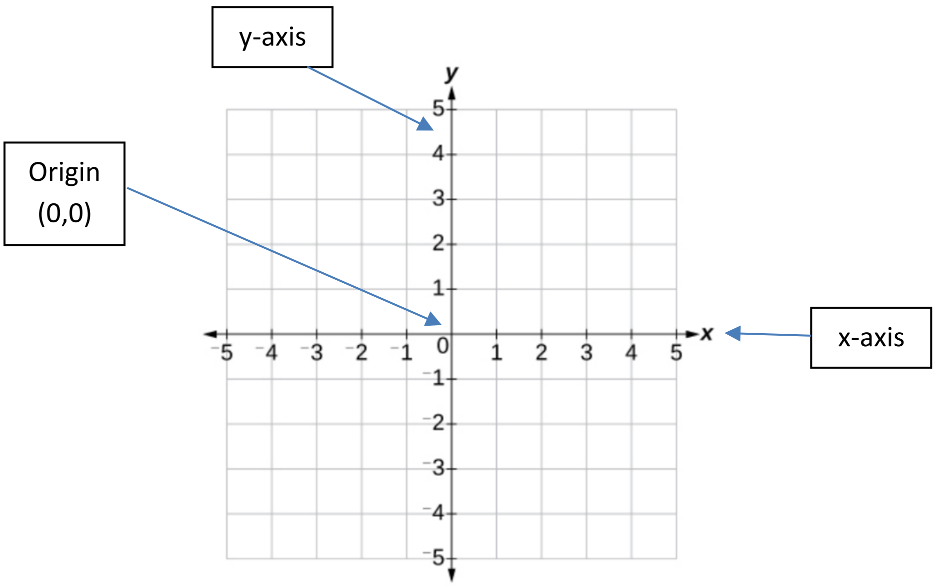
Practice Questions

1) $x = 2$	**2)** $x = 8$
3) $x = -0.29$	**4)** $x = 1.64$
5) $x = 8$	**6)** $x = -0.47$
7) $x = 1.25$	**8)** $x = 4$

Review Unit – Plotting Points

When we look at a graph, there is an x-axis and a y-axis. The x-axis is the horizontal axis (side to side) and the y-axis is the vertical axis (up and down).

The point where the x-axis and the y-axis cross is called the origin

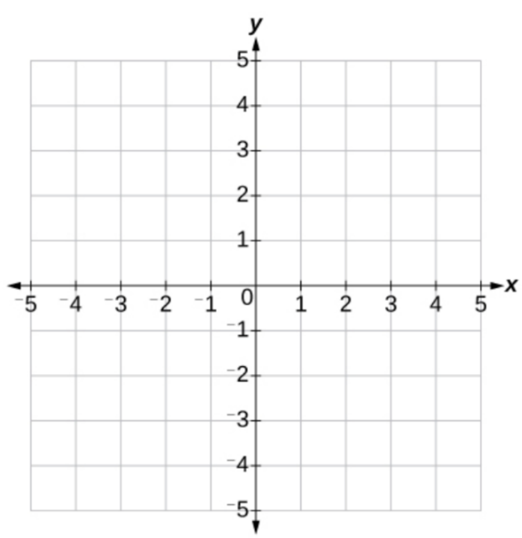


If we want to plot a point on a graph, we need to know the x-value (where the point is side to side) and the y-value (where the point is up and down).

We always write an ordered pair as (x, y). If we start at the origin $(0,0)$, the first number tells us how far to go left or right and the second number tells us how far to go up or down.

- If the first number is positive, move right
- If the first number is negative, move left
- If the second number is positive, move up
- If the second number is negative, move down

Ex: Plot the points
$A\ (3, 2)$
$B\ (-3, 2)$
$C\ (3, -2)$
$D\ (-3, -2)$

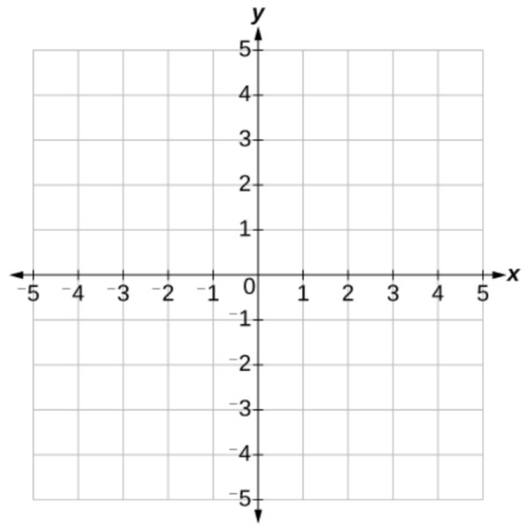


Review Unit – Plotting Points

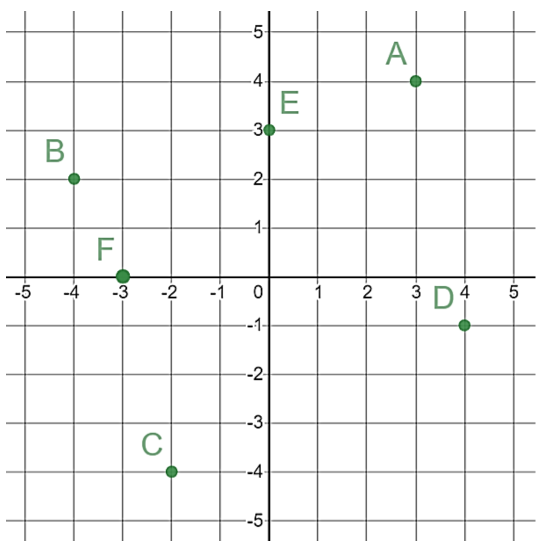
Try this question!

1) Plot the following points on the Cartesian plane below

$A\ (2, 5)$
$B\ (0, -3)$
$C\ (-3, 2)$
$D\ (4, 0)$
$E\ (-1, -4)$
$F\ (3, -2)$



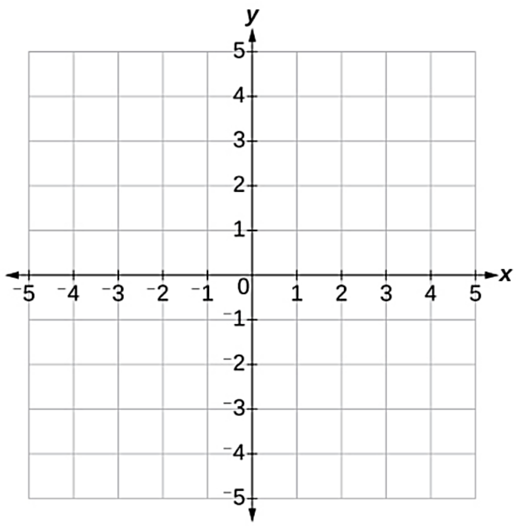
2) Write the ordered pair corresponding to each point on the graph below



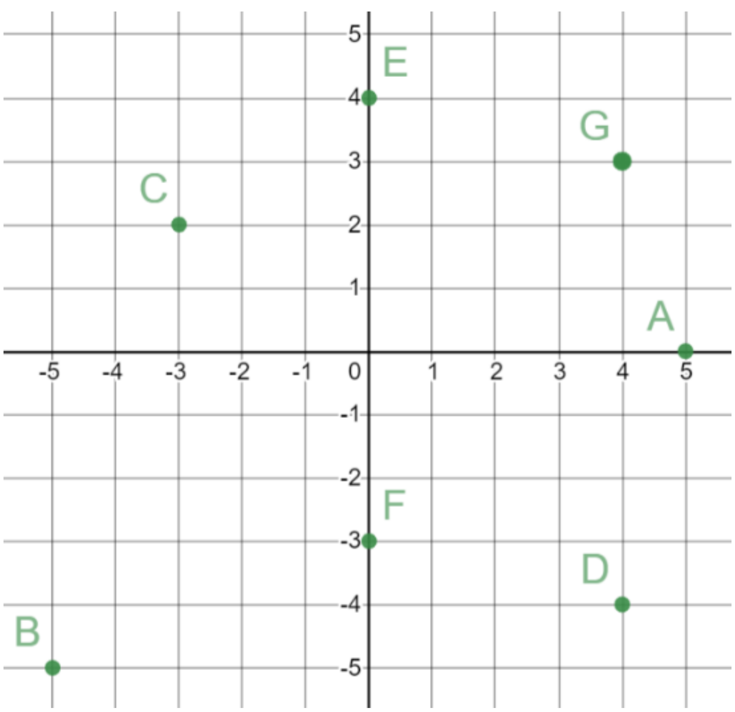
Practice Questions

1) Plot the following points on the Cartesian plane below

$A\ (2, 3)$
$B\ (-2, 3)$
$C\ (0, 1)$
$D\ (-3, 0)$
$E\ (-2, -1)$
$F\ (0, -2)$
$G\ (2, -1)$
$H\ (3, 0)$



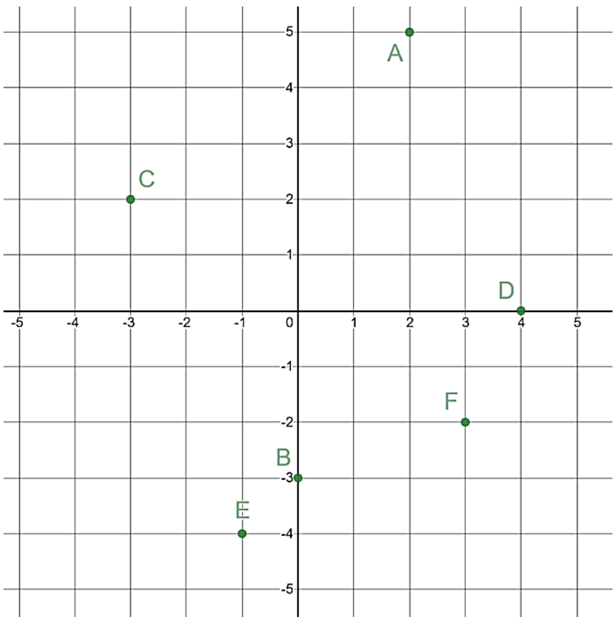
2) Write the ordered pair corresponding to each point on the Cartesian plane below



Review Unit – Plotting Points
Answer Key

Questions in the Notes

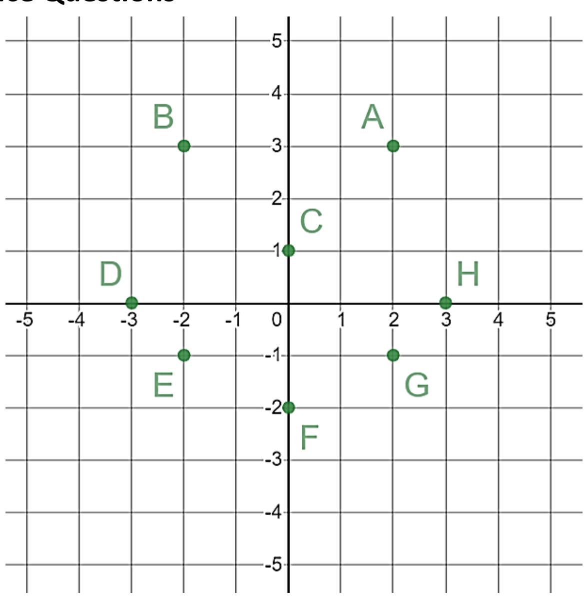
1)



2) $A\ (3, 4)$
$B\ (-4, 2)$
$C\ (-2, -4)$
$D(4, -1)$
$E\ (0, 3)$
$F\ (-3, 0)$

Practice Questions

1)



2) $A\ (5, 0)$
$B(-5, -5)$
$C\ (-3, 2)$
$D\ (4, -4)$
$E\ (0, 4)$
$F(0, -3)$
$G\ (4, 3)$

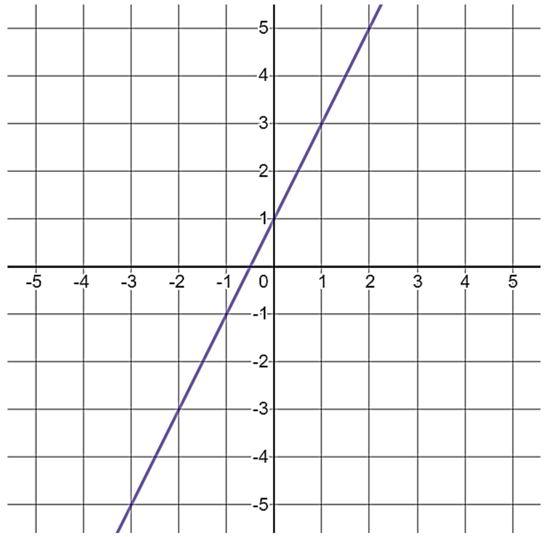
Systems of Equations Unit – Linear Functions

When we have a straight line, we can represent it using a table of values, a graph, or an equation (or rule).

Table of Values

x	y
-3	-5
-2	-3
-1	-1
0	1
1	3
2	5

Graph



Equation (or rule)

$y = 2x + 1$

The standard form for the rule of a linear function is $y = ax + b$

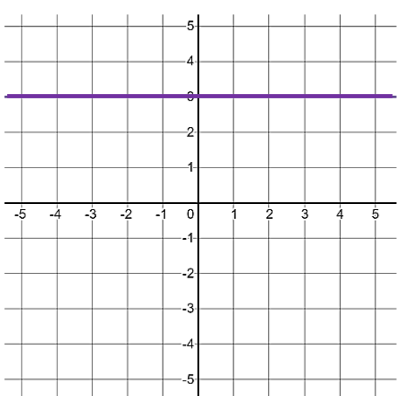
a is the rate of change (or slope)

b is the initial value (or y-intercept). It's where the line crosses the y-axis.

There are three different varieties of linear functions:

Constant Variation
(horizontal line)

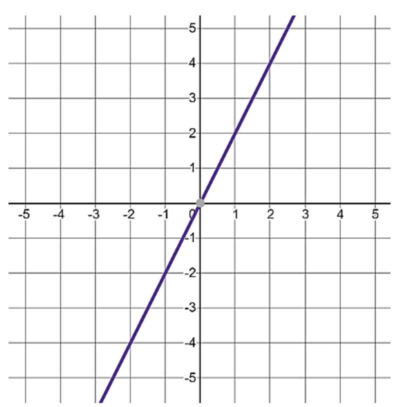
Ex:



$y = 3$

Direct Variation
(line goes through the origin, which means $b = 0$)

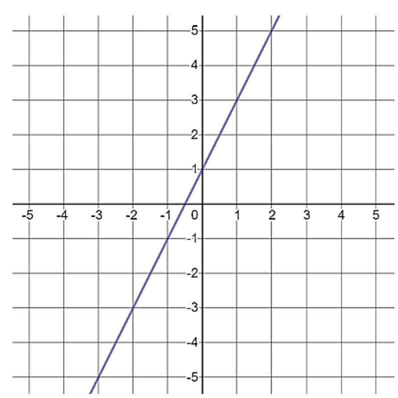
Ex:



$y = 2x$

Partial Variation
(line does not go through the origin, which means b can be any number except 0)

Ex:



$y = 2x + 1$

Systems of Equations Unit – Linear Functions

If we know the rule of a linear function, and are given a value for x, we can solve for y.

1) Write the rule
2) Replace x with the number given
3) Use algebra to solve for y

Ex: Given $y = 2x + 3$, find y when $x = 3$.

Try these questions!

1) Given $y = 3x - 1$, find y when $x = 2$.

2) Given $y = -2x + 3$, find y when $x = 4$.

3) Given $y = \frac{1}{2}x + 2$, find y when $x = 8$.

4) Given $y = \frac{2}{5}x - 4$, find y when $x = 15$.

Systems of Equations Unit – Linear Functions

If we know the rule of a linear function, and are given a value for y, we can solve for x.

1) Write the rule
2) Replace y with the number given
3) Use algebra to solve for x

Ex: Given $y = 2x + 5$, solve for x when $y = 15$.

Try these questions!

1) Given $y = 2x - 3$, find x when $y = 9$.

2) Given $y = -3x + 2$, find x when $y = 11$

3) Given $y = \frac{1}{2}x - 1$, find x when $y = 3$.

4) Given $y = \frac{2}{3}x + 7$, find x when $y = 11$.

Practice Questions

1) Given $y = 8x - 3$
a) Find y when $x = 2$
b) Find x when $y = 21$

2) Given $y = -2x + 10$
a) Find y when $x = -4$
b) Find x when $y = 30$

3) Given $y = \frac{1}{4}x + 3$
a) Find y when $x = 8$
b) Find x when $y = 6$

4) Given $y = -\frac{2}{3}x - 4$
a) Find y when $x = 9$
b) Find x when $y = -12$

Systems of Equations Unit – Linear Functions
Answer Key

Questions in the Notes

1) $y = 5$	**2)** $y = -5$	**3)** $y = 6$	**4)** $y = 2$
5) $x = 6$	**6)** $x = -3$	**7)** $x = 8$	**8)** $x = 6$

Practice Questions

1) a) $y = 13$
b) $x = 3$

2) a) $y = 18$
b) $x = -10$

3) a) $y = 5$
b) $x = 12$

4) a) $y = -10$
b) $x = 12$

Systems of Equations Unit – Finding the Rule of a Linear Function

The standard form for the rule of a linear function is

$$y = ax + b$$

The y is by itself on the left side

a is the rate of change (or slope)

b is the initial value (or y-intercept). It's where the line crosses the y-axis.

Ex: Find the rate of change an initial value for each of the following

$y = 3x + 4$ $\quad$ $y = -x + 1$ $\quad$ $2y - 4x = 6$

Try these questions!

1) $y = \frac{3}{4}x - 5$ $\quad$ **2)** $y = -2 + x$ $\quad$ **3)** $3x + 4y = 8$

If we are given 2 points on a line, we can find the rule of the line.

- Label one point (x_1, y_1). Label the other point (x_2, y_2).
- Find slope by using $a = \frac{y_2 - y_1}{x_2 - x_1}$
- Find initial value by using $b = y - ax$
- Write the rule by replacing a and b with the numbers you calculated above, and keep x and y as x and y.

If you are given more than 2 points, pick any two and ignore the others.

You can use either (x_1, y_1) or (x_2, y_2) for the x and y here. Replace y, a, and x with the numbers and solve for b.

Systems of Equations Unit – Finding the Rule of a Linear Function

Ex: Find the rule

a) of a line passing through the points $(1, 6)$ and $(3, 10)$

b) of a line passing through the points $(-2, 3)$ and $(6, -1)$

c) of a line that passes through the point (6, 12) and has a rate of change of 2.5

Try these questions!

Find the rule…

4) of a line passing through the points $(2, 7)$ and $(6, 9)$

5) of a line passing through the points $(-4, 5)$ and $(0, 2)$

6) of a line that passes through the point (1, -2) and has a rate of change of 3

Practice Questions

1) Find the rate of change and initial value of the following linear functions

a) $y = 2x + 3$

b) $y = \frac{5}{8}x - 4$

c) $y = -\frac{1}{2}x + 7$

d) $-3x + y = 12$

e) $2x = 3y + 6$

f) $7y + 6x = 35$

2) Find the rule…

a) of a line passing through the points $(2, 4)$ and $(5, -2)$

b) of a line passing through the points $(0, 3)$ and $(5, 7)$

c) of a line passing through the points $(0, -6)$ and $(3, 6)$

d) that passes through the point $(-4, 1)$ and has rate of change of 0.75

e) that as a rate of change of 1.8 and passes through the origin.

f) that has the following table of values

x	y
-2	-12
2	-4
6	4

Answer Key

Questions in the Notes

1) $a = \frac{3}{4}$

$b = -5$

2) $a = 1$

$b = -2$

3) $a = -\frac{3}{4}$

$b = 2$

4) $y = 0.5x + 6$

5) $y = -0.75x + 2$

6) $y = 3x - 5$

Practice Questions

1a) $a = 2$

$b = 3$

1b) $a = \frac{5}{8}$

$b = -4$

1c) $a = -\frac{1}{2}$

$b = 7$

1d) $a = 3$

$b = 12$

1e) $a = \frac{2}{3}$

$b = -2$

1f) $a = -\frac{6}{7}$

$b = 5$

2a) $y = -2x + 8$

2b) $y = 0.8x + 3$

2c) $y = 4x - 6$

2d) $y = 0.75x + 4$

2e) $y = 1.8x$

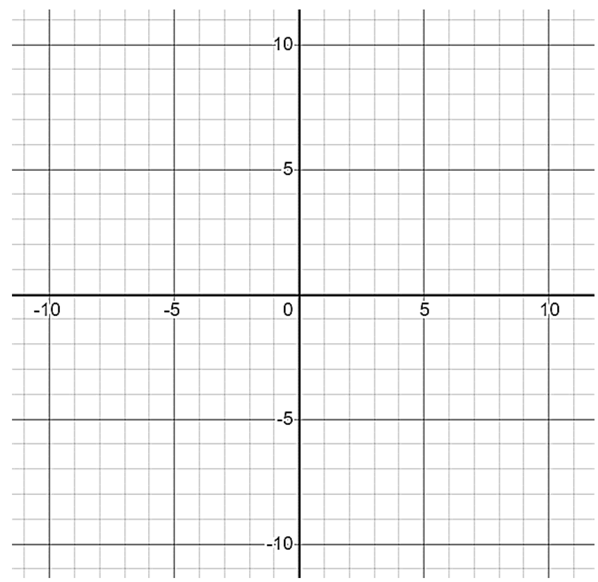
2f) $y = 2x - 8$

Systems of Equations Unit – Graphing a Linear Function

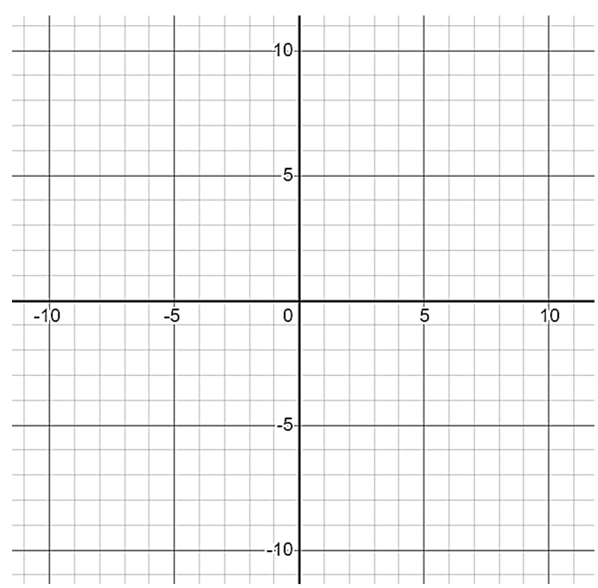
There are two methods to **graph a linear function**
- Using a table of values
- Using the rate of change and initial value

To use a table of values
- Pick 4 values of x and solve for their corresponding y values
- Plot the 4 points
- Connect the dots using a ruler

Ex: Graph the line $y = 2x + 3$ using a table of values



Try this question!
1) Graph the line $y = -2x + 5$ using a table of values



Systems of Equations Unit – Graphing a Linear Function

To use a rate of change and initial value

- Determine rate of change and initial value
- Plot the initial value at (0, b)
- From (0, b) use rate of change to plot another point
- Connect the dots using a ruler

Remember: $Rate\ of\ change = \frac{Rise}{Run}$

If rate of change is positive, rise up.

If rate of change is negative, rise down.

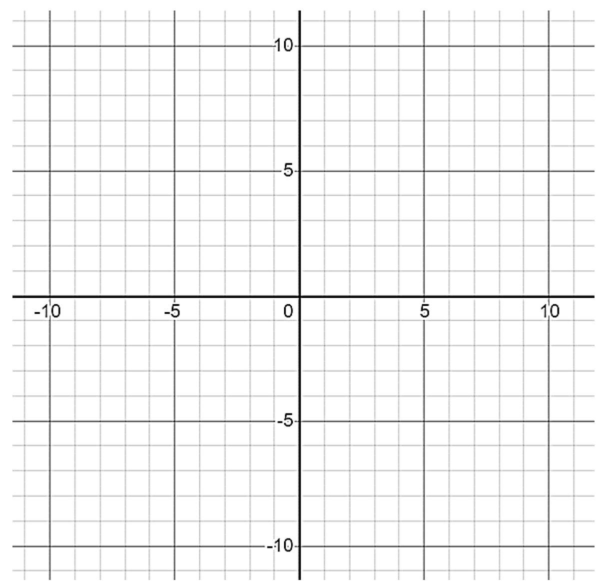
Always run right.

HINT!

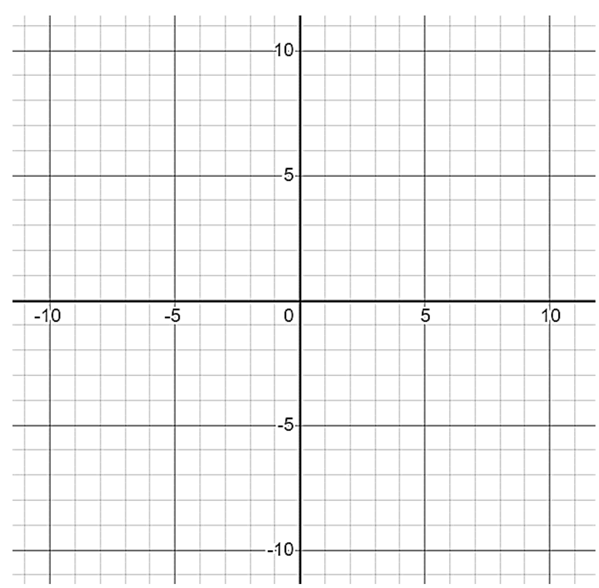
If your rate of change is not a fraction, you can turn it into one by dividing by 1.

Ex: If rate of change is 2, that can be written as $\frac{2}{1}$

Ex: Use rate of change and initial value to graph the line $y = \frac{2}{3}x - 5$

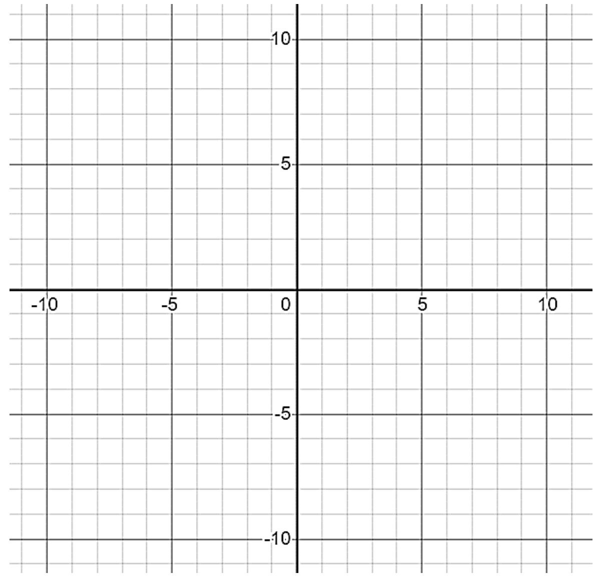


Ex: Use rate of change and initial value to graph the line $y = -3x + 8$

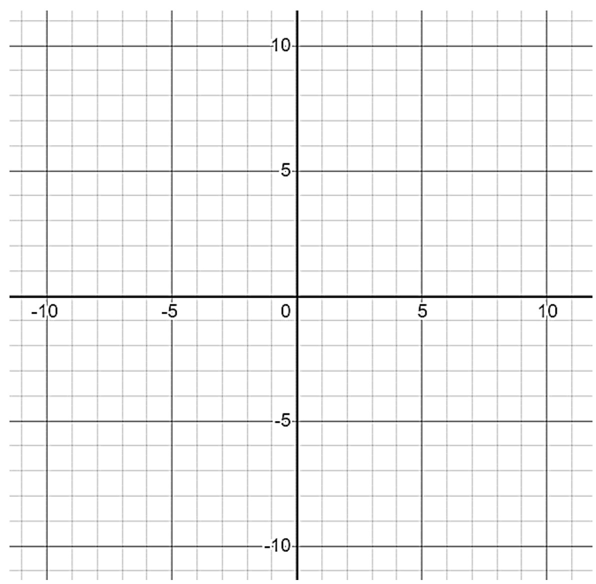


Try these questions!

2) Use rate of change and initial value to graph the line $y = \frac{1}{4}x + 1$



3) Use rate of change and initial value to graph the line $y = x - 4$



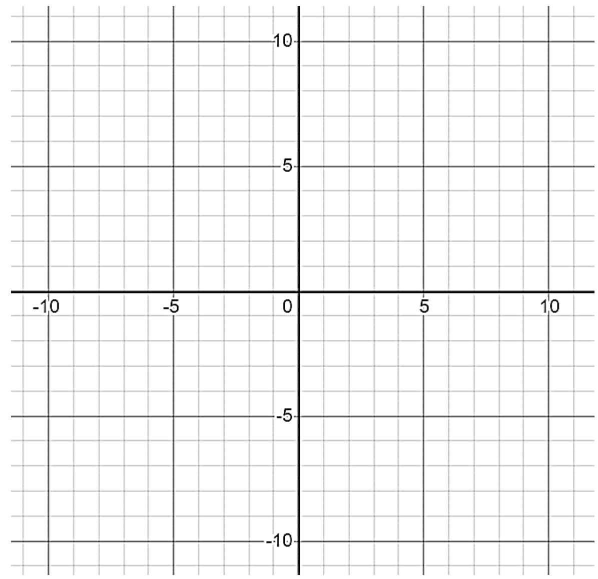
Practice Questions

1) Using a table of values, graph each of the following lines on the graph below

a) $y = 2x - 6$

b) $y = -\frac{3}{2}x + 8$

c) $y = -x$

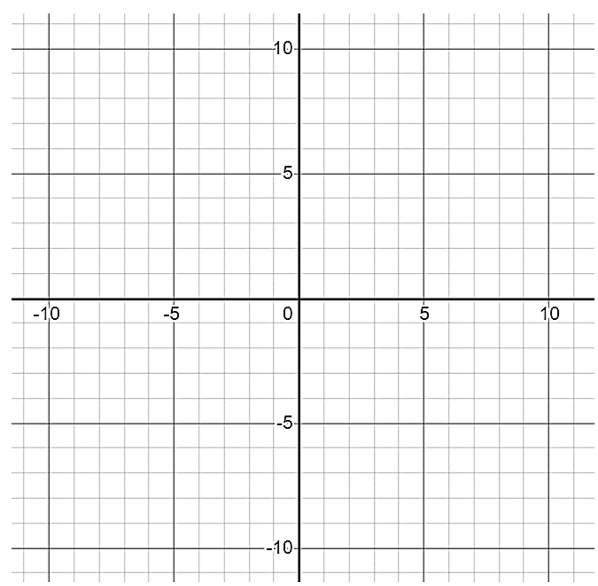


2) Using rate of change and initial value, graph each of the following lines on the graph below

a) $y = -\frac{3}{2}x + 10$

b) $y = 4x - 6$

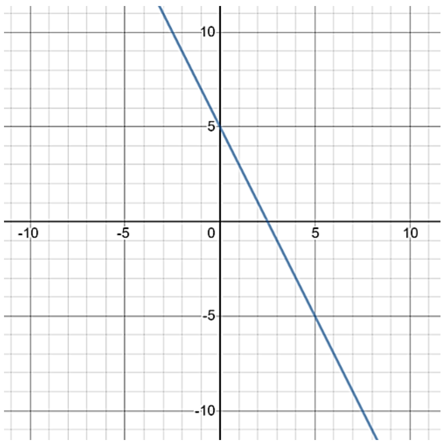
c) $y = x$



Systems of Equations Unit – Graphing a Linear Function
Answer Key

Questions in Notes

1)



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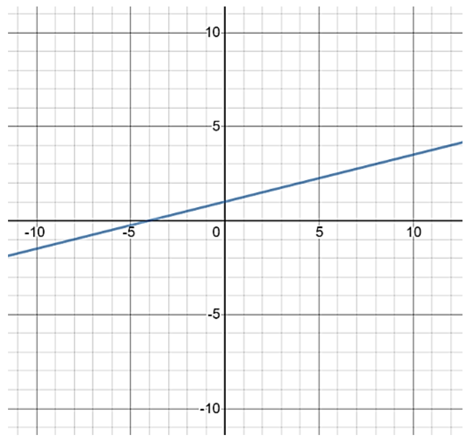
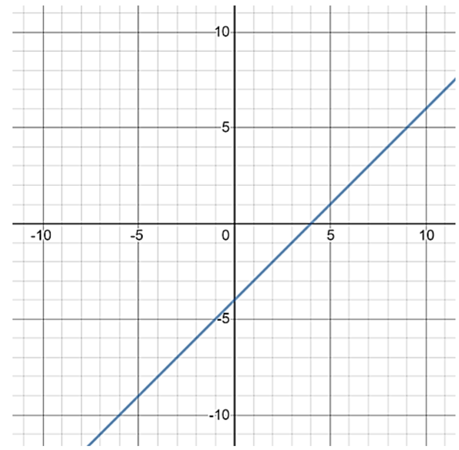
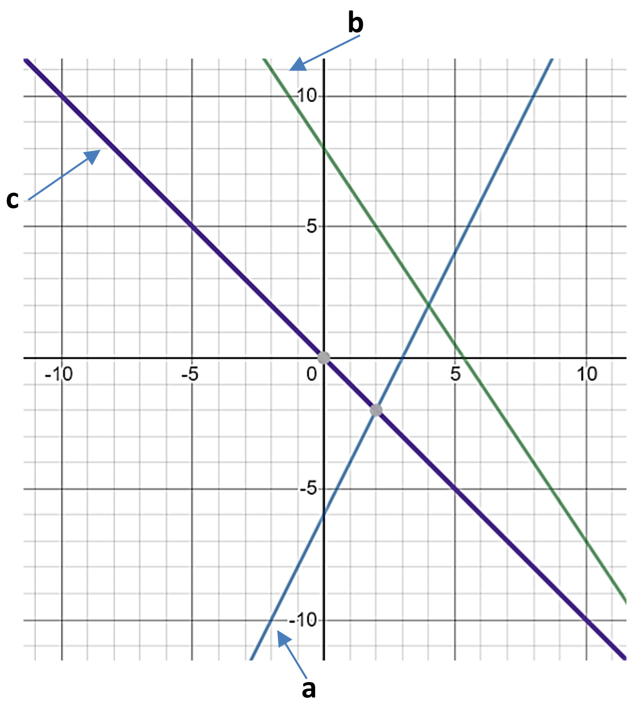


3)

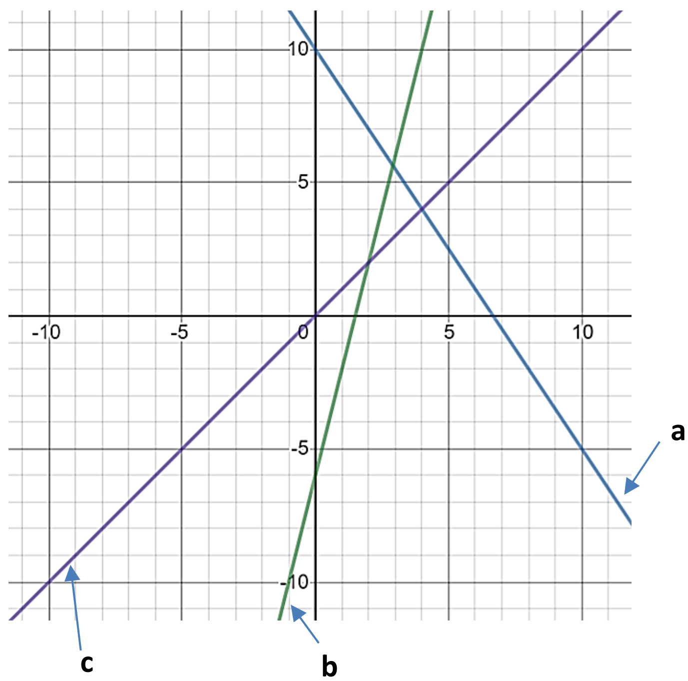


Practice Questions

1)



2)



Systems of Equations Unit – Systems of Equations

LINEAR SYSTEMS

A linear system is when we have two lines.

The **solution** is the point (x, y) where the two lines cross each other. We can find the solution to a system of equations graphically or algebraically (using elimination, comparison, or substitution).

We can check to see if a point (x, y) is a solution to the system by replacing the x and the y in both equations and seeing if it works.

Ex: Is $(2, 4)$ a solution to the following systems?

a) $y = x + 2 \; and \; y = -2x + 8$ $\quad$ $b)\; y = \frac{1}{2}x + 3 \; and \; y = 3x - 4$ $\quad$ c) $y = 6x - 2 \; and \; y = -4x - 3$

Try this question!

1) Is (3, 1) a solution to the following systems?

a) $y = \frac{2}{3}x - 1 \; and \; y = -x + 4$ $\quad$ b) $y = x - 2 \; and \; y = \frac{1}{3}x + 1$

Systems of Equations Unit – Systems of Equations

Practice Questions

1) Is (-3, 4) a solution to the following systems?

a) $y = x + 7 \ and \ y = -x + 2$

b) $4x + y = -8 \ and$
$2x + 3y = -6$

$c)$ $0.5x - 1.5y = -7.5 \ and$
$-3x + y = 13$

Systems of Equations Unit – Systems of Equations

Answer Key

Questions in Notes:

1a) yes **1b)** No

Practice Questions:

1)

a) no b) no c) yes

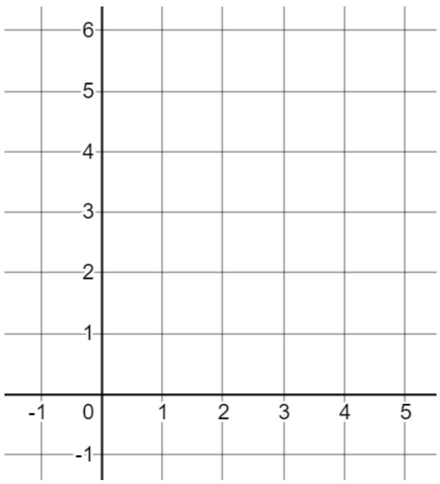
Systems of Equations Unit – Solving using Graphs

Using a graph to solve:

The find a solution, graph both lines. The solution is the point where the lines cross, written as an ordered pair (x, y). Remember, you may need to re-arrange the equation before you can graph it.

Ex: Find the solution to the linear system.

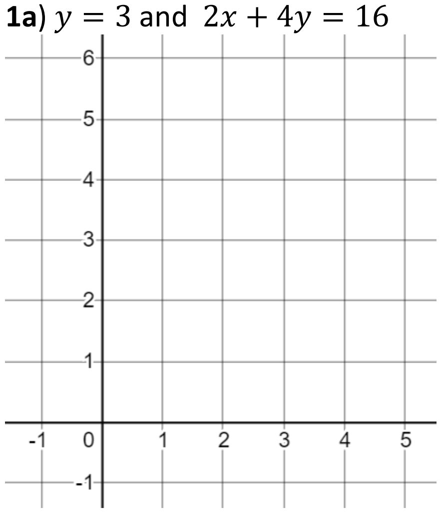
$y = 2x + 1$ and $x + y = 4$



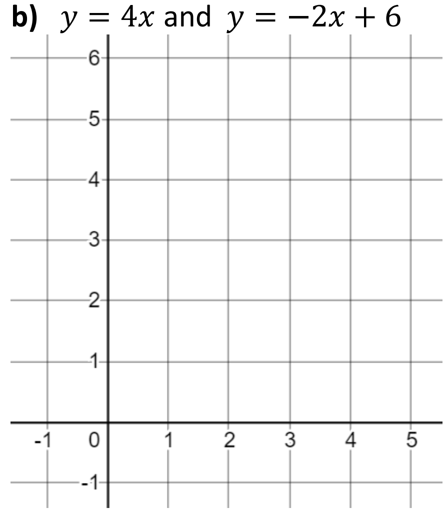
Try these questions!

Find the solution to the linear systems using graphs.

1a) $y = 3$ and $2x + 4y = 16$



b) $y = 4x$ and $y = -2x + 6$



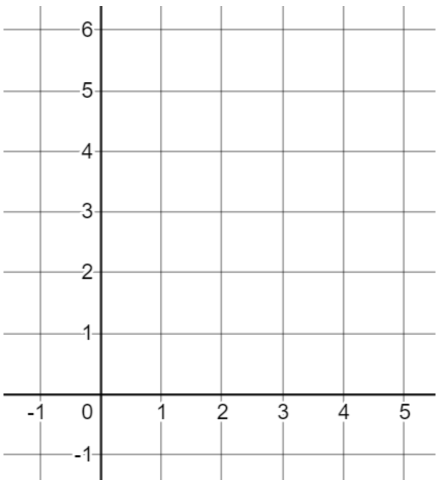
Systems of Equations Unit – Solving using Graphs

Practice Questions

Solve the following systems using graphs

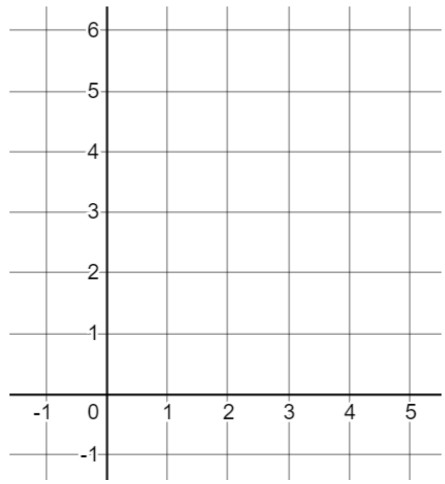
1)

$y = 4x - 10$ and $y = \frac{1}{3}x + 1$



2)

$y = -3x + 4$ and $y + 2 = 3x$

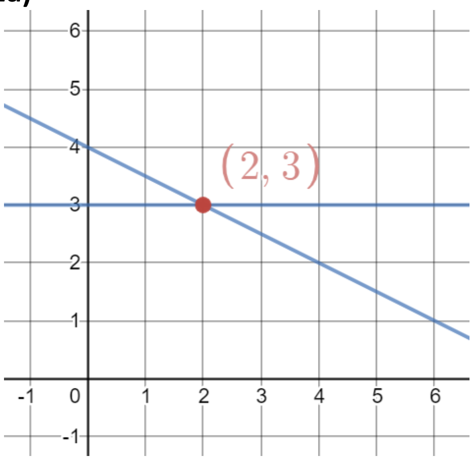


Systems of Equations Unit – Solving using Graphs

Answer Key

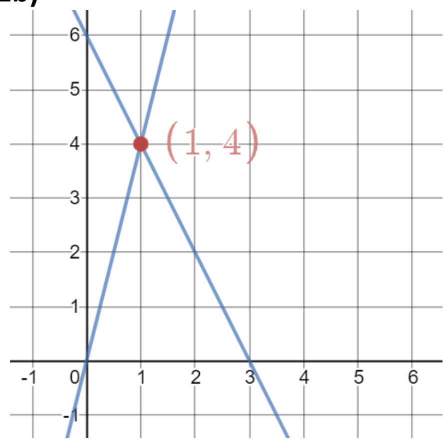
Questions in the Notes

1a)



The solution is $(2, 3)$

1b)



The solution is $(1, 4)$

Practice Questions

1) The solution is $(3, 2)$

2) The solution is $(1,1)$

Systems of Equations Unit – Solving using Elimination

The graphing method is not an accurate way to solve linear systems. For example, it is difficult to tell the different between $(2.3, 4.6)$ and $(2.2, 4.7)$. Therefore, we will use algebra to solve linear systems.

There are 3 methods we can use: elimination, comparison, and substitution.

Using the elimination method to solve:

- Both lines must be in the form $Ax + By = C$.
- Multiply the entire first equation by the coefficient of x in the second equation.
- Multiply the entire second equation by the coefficient of x in the first equation, but change the sign.
- Add the two equations.
- Solve for the remaining variable.
- Use the solution in either equation to solve for the other variable.
- Write the solution (x, y).

Ex: Find the solution to the linear systems.

a) $2x + 5y = 16$ and $3x - 4y = 1$

b) $4x - 5y = 10$ and $y = -\frac{5}{3}x + 35$

Systems of Equations Unit – Solving using Elimination

Try these questions!

Find the solution to the linear systems using elimination.

1a) $2x + 5y = -4$ and $3x - 2y = 13$

b) $3x + 4y = -6$ and $y = -2x + 1$

Practice Questions

1) Solve the system using elimination:
$8x - 6y = -20$ and $-16x + 7y = 30$

2) Solve the system using elimination:
$-4y - 11x = 36$ and $20 = -10x - 10y$

Systems of Equations Unit – Solving using Elimination
Answer Key

Questions in the Notes

1a) The solution is $(3, -2)$

1b) The solution is $(2, -3)$

Practice Questions

1) The solution is $(-1, 2)$

2) The solution is $(-4, 2)$

Systems of Equations Unit – Solving using Comparison

Using the comparison method to solve:

- Both lines must be in the form $y = ax + b$.
- Take the $ax + b$ pieces from each equation and set them equal to each other $ax + b = ax + b$.
- Solve for x.
- Use either equation (and the value of x you just found) to solve for y.
- Write the solution (x, y).

Ex: Find the solution to the linear systems using comparison.

a) $y = 2x + 1$ and $y = -1.5x + 4.5$ b) $y = -2x - 6$ and $5x + y = -3$

Try these questions!

Find the solution to the linear systems using comparison.

1a) $y = 2x + 5$ and $y = -4x + 11$ **1b)** $y = 0.5x + 2$ and $y - 2x = -1$

Systems of Equations Unit – Solving using Comparison

Practice Questions

1) Solve the system using comparison:
$y = x - 13$ and $y = -2x + 5$

2) Solve the system using comparison:
$y = -4x + 2$ and $x - y = 3$

Answer Key

Questions in the Notes

1a) The solution is $(1, 7)$

1b) The solution is $(2, 3)$

Practice Questions

1) The solution is $(6, -7)$

2) The solution is $(1, -2)$

Systems of Equations Unit – Solving using Substitution

Using the substitution method to solve:

- This method works best if we already know the value of x or y.
- Use the equation that has both variables and replace the known variable.
- Solve for the missing variable.
- Write the solution as (x, y).

Ex: Find the solution to the linear systems.

a) $x = 2$ and $y = 3x + 8$

b) $y = 3$ and $3x + 4y = 20$

Try these questions!

Find the solution to the linear systems using substitution.

1a) $y = 5$ and $y = 2x - 15$

1b) $x = 4$ and $3x + 2y = 20$

Practice Questions

1) Solve the system using substitution:
$y = -5$ and $5x + 4y = -20$

2) Solve the system using substitution:
$x = 3$ and $4x - y = 20$

Systems of Equations Unit – Solving using Substitution
Answer Key

Questions in the Notes

1a) The solution is $(10, 5)$

1b) The solution is $(4, 4)$

Practice Questions

1) The solution is $(0, -5)$

2) The solution is $(3, -8)$

Systems of Equations Unit – Variables and Rules

In order to solve a word, chart, or picture problem involving a system of equations, we must:

1) Identify the variables
2) Turns words, chart, or picture into 2 equations
3) Find the solution using comparison, elimination, or substitution
4) Answer the question

To start, let's focus on the first two steps.

Ex: Identify the variables and write the equations given the following scenario.
A cell phone company offers two different plans.

Plan A has a base fee of $10 plus an additional $0.15 per minute.
Plan B does not have a base fee but charges $0.25 per minute.

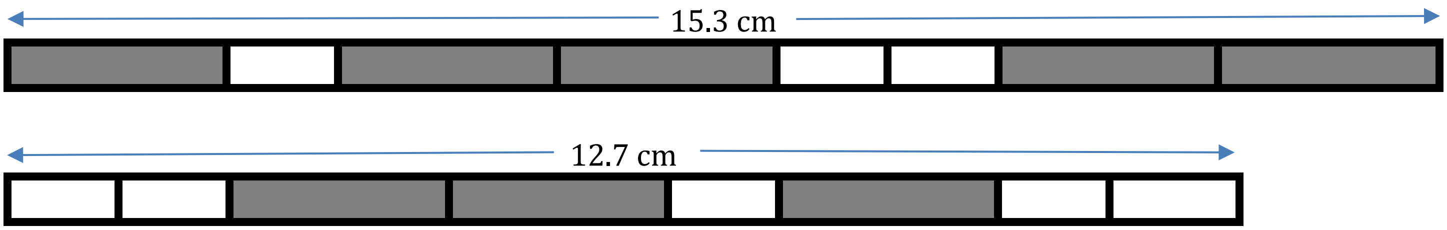
Ex: Identify the variables and write the equations given the following scenario.
Sam spends $14 buying 4 cans of pop and 3 bags of chips. The price of chips is triple the price of pop.

Ex: Identify the variables and write the equations given the following scenario.
The parking lot of a shopping center has small parking spaces and large parking spaces.
The table below provides information on two of the parking rows.

	Number of small spaces	Number of large spaces	Total width of the spaces in the row
Row 1	15	4	54.6m
Row 2	12	5	50.7m

Systems of Equations Unit – Variables and Rules

Ex: Identify the variables and write the equations given the following scenario.
The lengths of the white rectangles are all the same
The lengths of the dark rectangles are all the same



Try these questions!

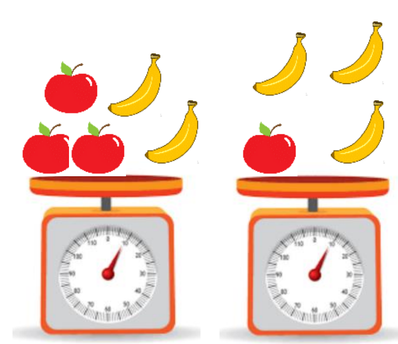
1) Identify the variables and write the equations given the following scenario.
Mrs. Payne wanted to treat the math department to sandwiches and coffee.

- The sandwiches cost $4 more than 2 times the cost of coffee.
- Mrs. Payne spent $121.50 and bought 25 cups of coffee and 12 sandwiches.

2) Identify the variables and write the equations given the following scenario.
A small theater is putting on a play. They sell adult tickets and youth tickets. The table below shows the numbers of each type of ticket sold and the total money the theater earned on two different days.

	Adult tickets sold	Youth tickets sold	Money earned
Day 1	8	4	$58
Day 2	12	5	$83.50

3) Identify the variables and write the equations given the following scenario.
The two scales below were used to weight apples and bananas. The weight of fruit on scale 1 is 25 units and the weight of fruit on scale 2 is 16.5 units.



Scale 1 Scale 2

Systems of Equations Unit – Variables and Rules

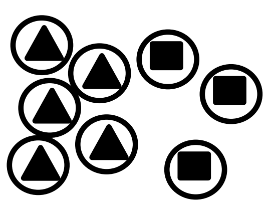
Practice Questions

1) Identify the variables and write the equations given the following scenario.
A piggy bank contains 15 coins. It only contains loonies and toonies. There is a total of $20 is the piggybank.

2) Identify the variables and write the equations given the following scenario.
A school is holding a two-day bake sale fundraiser. They are selling cookies and cupcakes. The results of the sale are presented in the table below.

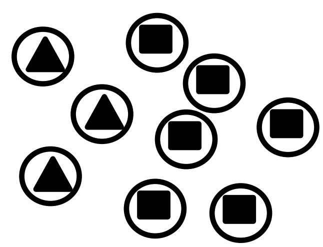
	Number of cookies sold	Number of cupcakes sold	Total profit
Day 1	20	15	$50
Day 2	10	25	$60

3) Identify the variables and write the equations given the following scenario.
A game of chance involves picking a handful of coins, each with a different shape drawn on one side. Riley and Mackenzie both play this game and the results are shown below.



Riley

21 points



Mackenzie

19 points

Systems of Equations Unit – Variables and Rules
Answer Key

Questions in Notes

1)
x: # of sandwiches bought
y: # of cups of coffee bought

$x = 2y + 4$
$25x + 12y = 121.50$

2)
x: # of adults tickets sold
y: # of youth tickets sold

$8x + 4y = 58$
$12x + 5y = 83.50$

3)
x: # of apples
y: # of bananas

$3x + 2y = 25$
$1x + 3y = 16.5$

Practice Questions

1)
x: # of loonies
y: # of toonies

$x + y = 15$
$1x + 2y = 20$

2)
x: # of cookies sold
y: # of cupcakes sold

$20x + 15y = 50$
$10x + 15y = 60$

2)
x: # of triangle coins
y: # of rectangle coins

$5x + 3y = 21$
$3x + 5y = 19$

Systems of Equations Unit – Solving Word, Chart, and Picture Problems

Recall: in order to solve a word, chart, or picture problem involving a system of equations, we must:

1) Identify the variables
2) Turns words, chart, or picture into 2 equations
3) Find the solution using comparison, elimination, or substitution
4) Answer the question

Ex: Solve the word problem below.
Mike buys 3 juice boxes and 2 muffins for $6. John buys 2 juice boxes and 4 muffins for $8. How much does David have to spend in order to buy 5 juice boxes and 1 muffin?

Try this question!

1) Students are holding a car wash to raise money for their class trip. On the first day they earn $350 by washing 20 cars and 10 trucks. On the second day they earn $475 washing 25 cars and 15 trucks. How much would the students earn if they washed 30 cars and 18 trucks?

Systems of Equations Unit – Solving Word, Chart, and Picture Problems

Ex:

For the past two years, Philemon Wright has sold t-shirts and hoodies to the Grade 10 students. The sales and total cost are presented in the table below.

	Number of t-shirts sold	Number of hoodies sold	Total Cost
Year 1	130	80	$3950
Year 2	110	100	$4150

This year the cost of hoodies increased by $5. If the school sells 100 t-shirts and 120 hoodies, what will be the total cost?

Try this question!

2) Two families went to a water park. The price of admission can be found in the table below.

	Number of youth tickets	Number of adult tickets	Total cost
Anderson Family	3	2	$110
Beck Family	2	4	$140

The Connor family would also like to go to the water park, but only have $110 to spend on the outing. They would need to buy 4 youth tickets and 1 adult ticket. Do they have enough money saved to go to the water park?

Systems of Equations Unit – Solving Word, Chart, and Picture Problems

Ex: Christy and Julie are playing with toy cars and toy trucks. They each line up a combination of cars and trucks. Each toy car is the same length and each toy truck is the same length. Christy's line of vehicles is 47 cm. Julie's line of vehicles is 66 cm.

Christy's line

Julie's line

How long would the line of vehicles be if Lauren used 6 cars and 7 trucks?

Try this question!

3) A zoo is planning on bringing in a new baby elephant and a new baby giraffe that were rescued. The animals will arrive on a truck, but the zoo needs to ensure the truck can move all the animals in one trip. The zoo has previously weighed baby animals and the results are shown in the image below.

The truck can carry up to 575 lbs. Will the zoo be able to move the baby elephant and the baby giraffe?

Systems of Equations Unit – Solving Word, Chart, and Picture Problems

Practice Questions

1) A gardener grows roses and tulips. There are a total of 300 plants in the garden. There are twice as many tulips as roses. How many roses are there in the garden?

2) John collects stuffed dinosaurs and stuffed bears. He has 20 animals in his collection. Each stuffed bear is worth $10, each stuffed dinosaur is worth $15 and the total value of his collection is $260. If the value of the stuffed bears doubled, how much would John's collection be worth?

3) Two friends, Naomi and Kendra, decide to go apple picking. The number of apples and the cost for each friend is presented in the table below.

	Number of McIntosh apples	Number of Red Delicious apples	Total Cost
Naomi	5	8	$7.40
Kendra	6	4	$5.80

A third friend, Kiera, is planning on going apple picking next weekend and would like to pick 8 McIntosh apples and 3 Red Delicious apples. How much will Kiera have to spend?

4) Every year the school band holds a fundraiser for their annual trip to Banff. They sell tomato plants and pepper plants. Daniel is in the band and his sales from previous years are presented in the table below.

	Number of tomato plants sold	Number of pepper plants sold	Total amount earned
Grade 8	10	8	$44
Grade 9	15	20	$90

Daniel wants to earn $116 this year and knows he will sell 25 tomato plants. How many pepper plants will he need to sell?

5) A construction company is building houses and they want to plant a row of trees behind each house. They have previously planted the following rows of trees.

Cost: $360

Cost: $460

The company would like to plant the following row of trees, but only has a budget of $490.

Has the company budgeted enough money to plant this row of trees?

Systems of Equations Unit – Solving Word, Chart, and Picture Problems
Answer Key

Questions in Notes
1) The students would make $570.

2) Yes, the Connor family has enough money saved to go to the water park.

3) Yes, the zoo will be able to move the baby elephant and the baby giraffe.

Practice Questions
1) There are 100 roses in the garden.

2) John's collection would be worth $340 if the value of the stuffed bears doubled.

3) Kiera will have to spend $6.45

4) Daniel will need to sell 22 pepper plants.

5) No, the company has not budgeted enough money to plant the row of trees.

Analytic Geometry Unit – Distance

To find the distance between any two known points, we can use the **distance** formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

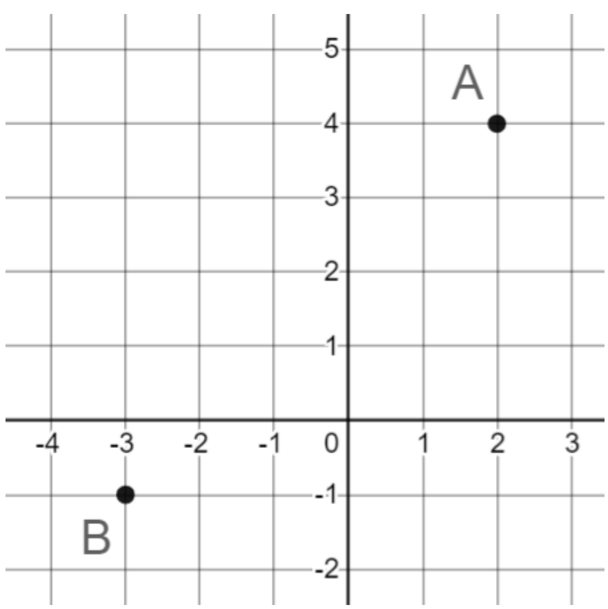
Just like when finding rate of change, label one point (x_1, y_1) and label the other point (x_2, y_2). It does not matter which point is which.

Ex: Find the distance between the points $P(4, -1)$and $Q(-2, -3)$.

Try these questions!

1) Find the distance between the points $M(-2, 5)$and $N(-2, -1)$

2) Find the distance between the points A and B.



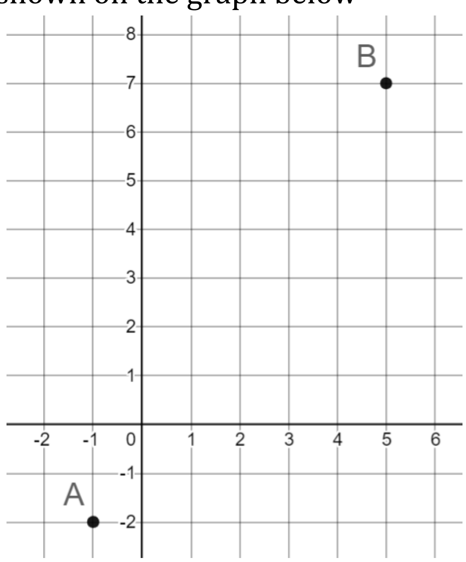
Analytic Geometry Unit – Distance

Practice Questions

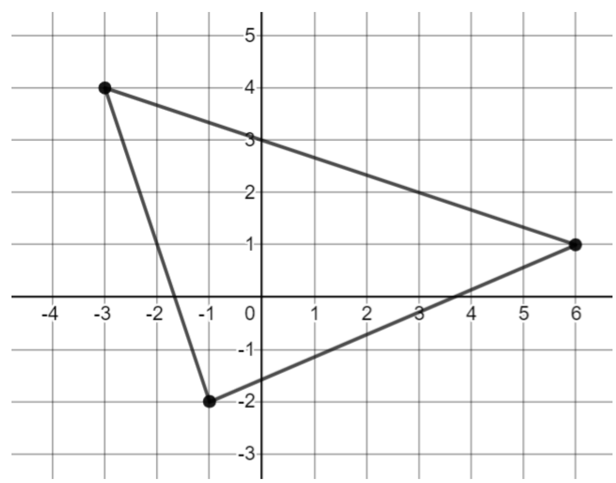
1) Find the distance between the points $A(3,4)$ and $B(8,6)$

2) Find the distance between the points $A(-4,-10)$ and $B(-52,-112)$

3) Find the distance between the two points shown on the graph below



4) Find the perimeter of the triangle shown on the graph below



Analytic Geometry Unit – Distance
Answer Key

Questions in Notes
1) 6 units

2) 7.071 units

Practice Questions
1) 5.3852 units

2) 112.7298 units

3) 10.8167 units

4) 23.4272 units

Analytic Geometry Unit – Midpoint

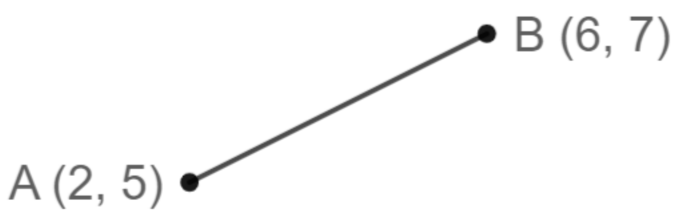
If we know two points, we can also find the **midpoint**. That is, we can find the point that is half way between the two known points.

Just like when finding rate of change and distance, label one point (x_1, y_1) and label the other point (x_2, y_2). It does not matter which point is which.

To find the midpoint

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Ex: Find the midpoint between points A and B shown below



Try these questions!

1) Find the midpoint between $A(2, 5)$ and $B(6, 7)$

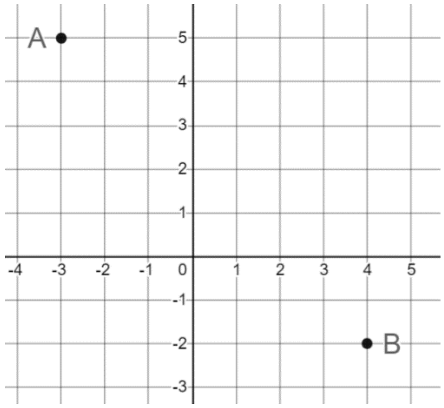
2) Find the midpoint between $C\left(\frac{1}{2}, -5\right)$ and $D(-3, 2)$

Hint: Turn the fraction $\frac{1}{2}$ into a decimal before using the midpoint formula

Analytic Geometry Unit – Midpoint

Practice Questions

1) Find the midpoint between points A and B shown in the graph below



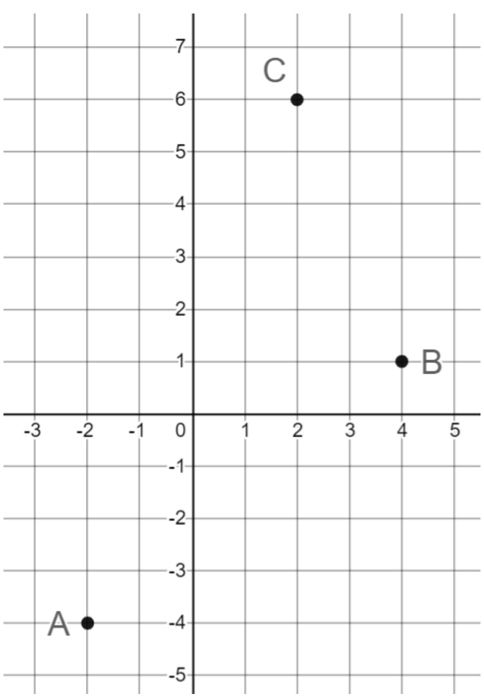
2) Find the midpoint between the points $A(-3, 5)$ and $B(-6, -1)$

3) Use the graph below to answer the following questions

a) Determine the midpoint of $\overline{AB}$

b) Determine the midpoint of $\overline{BC}$

c) Determine the distance between the midpoint of $\overline{AB}$ and the midpoint of $\overline{BC}$



Analytic Geometry Unit – Midpoint
Answer Key

Questions in Notes

1) $(4, 6)$

2) $(-1.25, -1.5)$

Practice Questions

1) $(0.5, 1.5)$

2) $(-4.5, 2)$

3) a) $(1, -1.5)$

b) $(3, 3.5)$

c) 5.3852 units

The midpoint formula is used to determine a point located halfway between two other points.

To find a point located anywhere between two points, we use the **division point** formula.

When we are using division point, the first point (the starting point) must be labeled (x_1, y_1) and the second point (the ending point) must be labeled (x_2, y_2)

There are two division point formulas depending on how the question is asked:

If the question uses the word "ratio":

$$P = (x_1 + \frac{a}{a+b}(x_2 - x_1), y_1 + \frac{a}{a+b}(y_2 - y_1))$$

If the question does not use the word "ratio":

$$P = (x_1 + \frac{a}{b}(x_2 - x_1), y_1 + \frac{a}{b}(y_2 - y_1))$$

a and b are given by the fraction or ratio you want to go from one point to the other

Ex: Find the coordinates of point P located at a ratio of $\frac{1}{2}$ from $A\ (1, 3)$ to $B\ (4, 9)$

Ex: Find the coordinates of point P located $\frac{1}{4}$ of the way from $A\ (-2, 4)$ to $B\ (2, 12)$

Analytic Geometry Unit – Division Point

Try these questions!

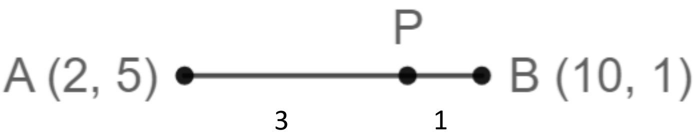
1) Find the coordinates of point P, which is located at a ratio of $\frac{1}{3}$ from point $A\ (2,4)$ to point $B\ (10,0)$

2) Find the coordinates of point P, which is located at a ratio of 2:5 from point $A\ (1,7)$ to point $B\ (-4,2)$

3) Find the coordinates of point P, which is located $\frac{3}{4}$ of the way from point $A\ (2,7)$ to point $B\ (12,3)$

Analytic Geometry Unit – Division Point

Ex: Given the following diagram, find the coordinates of point P



Note: Point P is located such that it divides $\overline{AB}$ into a ratio of $\frac{3}{1}$

OR

Point P is located $\frac{3}{4}$ of the way from A to B

Ex: Given the diagram below shows 3 congruent (equal) segments, find the coordinates of points F and G

Try this question!

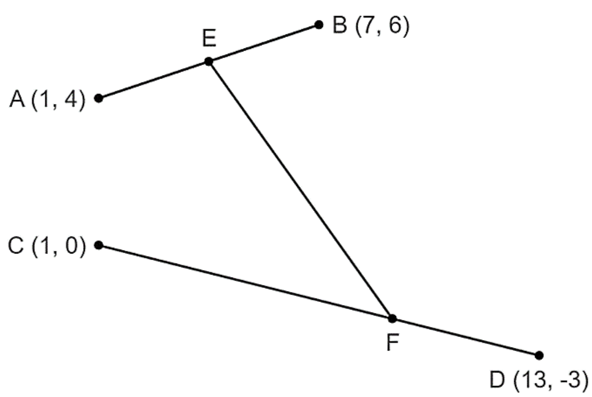
4) Given the diagram below shows 3 congruent (equal) segments, find the coordinates of points P

Practice Questions

1) Find the coordinates of point P, which is located $\frac{3}{4}$ of the way from point $A\ (-4, 3)$ to point $B\ (8, -13)$

2) Find the coordinates of point P, which is located at a ratio of $\frac{1}{4}$ of the way from point $A\ (5, 9)$ to point $B\ (15, 44)$

3) In the diagram below, point E is the midpoint of $\overline{AB}$ and point F is located $\frac{2}{3}$ of the way from point C to point D. Find the length of $\overline{EF}$.



Analytic Geometry Unit – Division Point
Answer Key

Questions in the Notes
1) $(4, 3)$

2) $(-0.4286, 5.5714)$

3) $(9.5, 4)$

4) $(0, -1)$

Practice Questions
1) $(5, -9)$

2) $(7, 16)$

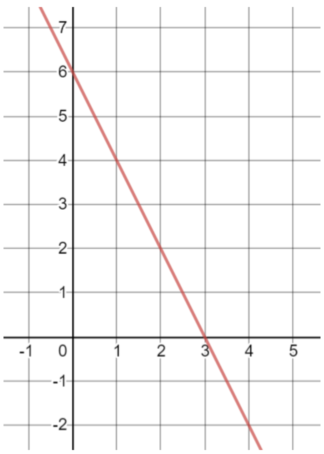
3) 8.6023 units

The **x-intercept** is the point where a line crosses the x-axis. It is located at $(x, 0)$. It is also called the "zero".

We can find the x-intercept by:

- looking at a graph to see where the line crosses the x-axis
- using the rule of the line, replacing y with 0, and solving for x

Ex: Find the x-intercept

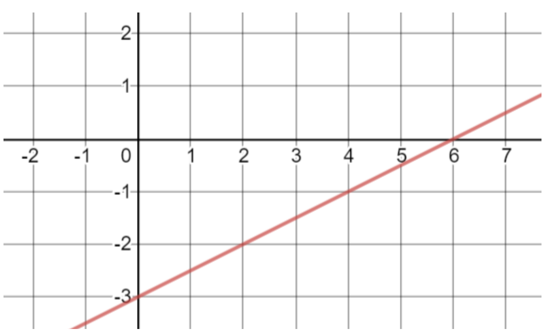


Ex: Find the x-intercept of the line $y = 2x - 8$

Try these questions!

1) Find the x-intercept for each line below

a)



b) $y = -3x - 12$

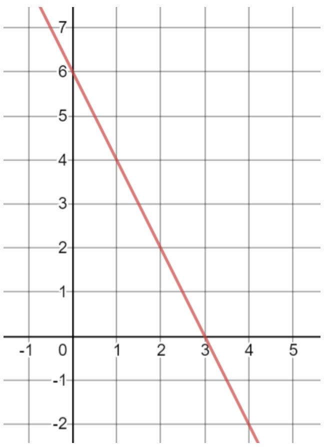
Analytic Geometry Unit – Intercepts

The **y-intercept** is the point where a line crosses the y-axis. It is located at $(0, y)$. It is also called the "initial value". The y-intercept is b in the standard form of a line: $y = ax + b$

We can find the y-intercept by:

- looking at a graph to see where the line crosses the y-axis
- using the rule of the line:
    - replacing y with 0, and solving for x
    - if the line is already in $y = ax + b$ form, then the y-intercept is b.

Ex: Find the y-intercept



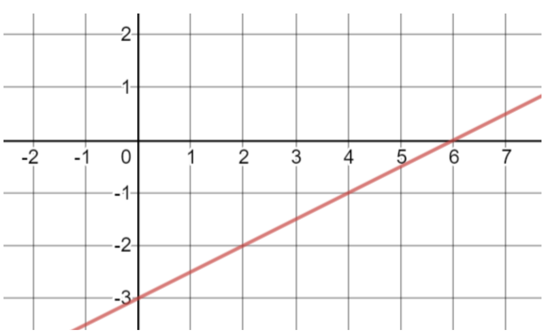
Ex: Find the y-intercept

$2x + 3y = 9$

Try these questions!

1) Find the y-intercept for each line below

a)



b) $2y = -3x - 12$

Analytic Geometry Unit – Intercepts

If we know the intercepts, we can also find the rule of the line.

Remember the x-intercept is the point $(x, 0)$ and the y-intercept is the point $(0, y)$

Ex: Find the rule of a line that has an x-intercept of 6 and a y-intercept of -2

Try this question!

3) Find the rule of a line that has an x-intercept of 2 and a y-intercept of -8

Practice Questions

1) Find the x-intercept and the y-intercept for each of the following functions:

a) $y = 3x - 4$

b) $y = 0.5x + 5$

c) $5x + 15y = 45$

d) $3y = 12x + 24$

2) Find the rule of a line that has an x-intercept of -8 and a y-intercept of 4

3) Find the rule of a line that has a y-intercept of 3 and a slope of 8

4) Find the rule of a line that has an x-intercept of 6 and a slope of 3

Answer Key

Questions in Notes

1a) x-intercept is 6

1b) x-intercept is -4

2a) y-intercept is -3

2b) y-intercept is -6

3) $y = 4x - 8$

Practice Questions

1a) x-intercept is $\frac{4}{3}$ $(or\ 1.333)$ y-intercept is -4

1b) x-intercept is -10 y-intercept is 5

1c) x-intercept is 9 y-intercept is 3

1d) x-intercept is -2 y-intercept is 8

2) $y = 0.5x + 4$

3) $y = 8x + 3$

4) $y = 3x - 18$

Analytic Geometry Unit – Parallel Lines

Two lines are **parallel** if they never cross each other.

Parallel lines

- always have the same rate of change (a)
- do not need to have the same initial value (b)

There are two types of parallel lines:

- Parallel and Distinct lines have the same a values but different b values
- Parallel and Coincident lines have the sae a values and the same b values (if we were to graph the lines, one line would be on top of the other line).

Ex: Are the two lines parallel?

$y = 2x + 3$	$y = 3x - 2$	$2x + 3y = 6$
$y = 2x - 4$	$y = 2x - 3$	$4x = -6y + 12$

Try this question!

1) Label each of the following pairs of lines as: parallel and distinct, parallel and coincident, or not parallel.

$2y = 3x + 8$	$-6x = -2y - 8$	$2x + 4y = 2$
$6x = 4y - 16$	$y = 3x + 8$	$2x = 4y + 2$

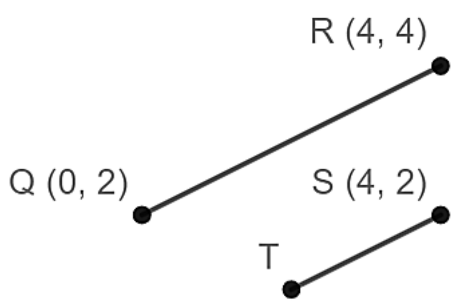
If we know the rule of one line, we can find the rule of a **parallel line** if we know one point that the parallel line passes through because the rates of change have to be the same in both rules.

Ex: Find the rule of a line parallel to $y = 5x + 2$ that passes through the point $P\ (1, 4)$

Ex: Consider the diagram below:

- Point Q and Point R are on the line segment QR
- Point s is found on line segment ST
- Line segment QR is parallel to line segment ST

What is the rule of line segment ST?



Try these questions!

2) The rule of line 1 is $4x = 2y + 12$. Line 2 is parallel to line 1 and passes through the point $(2, 8)$. What is the rule of line 2?

3) Line 1 passes through the points (2, 5) and (6, 7). Line 2 is parallel to line 1 and has an x-intercept of 6. What is the rule of line 2?

Practice Questions

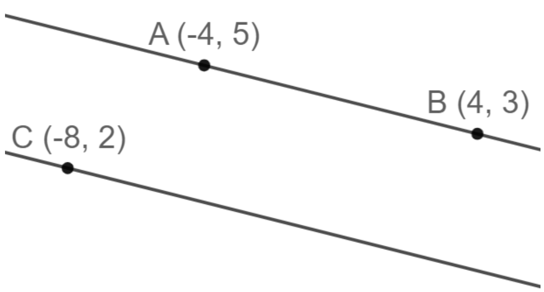
1) Are the pairs of line parallel?

a) $y = 5x + 2 \text{ } and \text{ } y = 2 + 5x$ b) $y = -x + 1 \text{ } and \text{ } -x + y = 4$ c) $2x + 4y = 20 \text{ } and \text{ } 6x + 12y = -24$

2) Find the rule of a line parallel to the line $y = -3x + 1$ and passing through $Q\ (5, -16)$

3) Find the rule of a line parallel to the line $3x + 2y = 12$ and passing through $P\ (2, 3)$

4) The lines below are parallel. Find the rule for the line passing through point C.



Answer Key

Questions in Notes

1a) parallel and coincident

1b) parallel and distinct

1c) not parallel

2) $y = 2x + 4$

3) $y = 0.5x - 3$

Practice Questions

1a) yes

1b) no

1c) yes

2) $y = -3x - 1$

3) $y = -\frac{3}{2}x + 6$

4) $y = -0.25x$

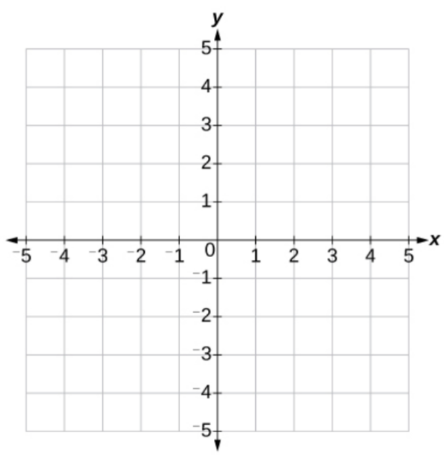
Two lines are **perpendicular** if they cross each other at right angles. The rates of change of perpendicular lines are negative reciprocals.

Negative reciprocal means we flip the fraction and change the sign.

Ex: Write the negative reciprocal of each of the following:

$\frac{3}{4}$ $\qquad -\frac{5}{8}$ $\qquad \frac{1}{2}$ $\qquad -6$

Ex: Find the rates of change for each line graph both lines. $y = -2x + 1 \; and \; y = \frac{1}{2}x + 3$



Sometimes we will have the rate of change written as a decimal instead of a fraction. In that case, you can tell if two lines are perpendicular by multiplying their rates of change. Two lines are perpendicular if the product of their rates of change is -1.

Ex: Are the following lines perpendicular?

a) $y = \frac{2}{3}x + 4$

$y = -\frac{3}{2}x - 8$

b) $2x + 3y = 7$

$3x - 2y = 10$

c) $y = 4x + 2$

$y = -0.25x - 8$

d) $2x + 7y = 8$

$3.5x = y - 12$

Analytic Geometry Unit – Perpendicular Lines
Try these questions!

Are the following lines perpendicular?

1a)

$$y = \frac{1}{2}x + 3$$

$$y = -\frac{1}{2}x + 5$$

1b)

$$y = \frac{2}{5}x + 1$$

$$y = \frac{5}{2}x - 4$$

1c)

$$y = 0.125x - 2$$

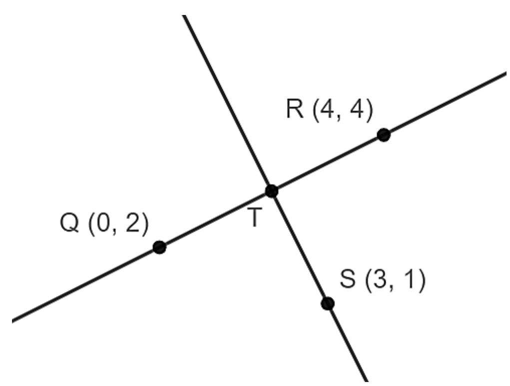
$$y = -8x - 1$$

If we know the rule of one line, we can find the rule of a **perpendicular line** if we know one point that the perpendicular line passes through because the rates of change have to be negative reciprocals.

Ex: Find the rule of a line passing through the point $L\ (2, 6)$ and perpendicular to the line passing through $P\ (-2, 1)$ and $Q\ (3, -1)$

Ex: Consider the following diagram

- Point Q and point R are on line segment QR
- Point S is found on line segment ST
- Line segments QR and ST are perpendicular.



What is the rule of line segment ST?

Try these questions!

2) Are these lines perpendicular?

a)

$$y = \frac{3}{2}x + 2$$

$$y = -\frac{2}{3}x + 8$$

b)

$$y = 4x + 8$$

$$y = -4x + 2$$

3) Find the rule of a line perpendicular to $y = \frac{2}{3}x - \frac{8}{5}$ that passes through the point $P\ (2, 6)$

4) Find the rule of a line perpendicular to $6x - 2y = 8$ that passes through the point $P\ (-3, 0)$

Practice Questions

1) Are the following pairs of lines parallel, perpendicular, or neither?

a)

$y = -\frac{1}{7}x + 2$

$y = 7x + 1$

b)

$4x + 5y = 25$

$10x - 8y = 16$

c)

$y = 3x - 1$

$y = -3x + 4$

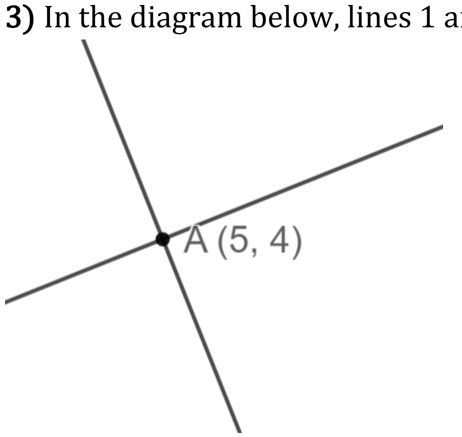
d)

$y = 0.4x + 1$

$-2x + 5y = 30$

2) Find the rule of a line perpendicular to $x - 4y = 8$ that passes through the point $P(1, -2)$

3) In the diagram below, lines 1 and 2 are perpendicular. Find the rule of line 2.



Answer Key

Questions in Notes

1a) no

1b) no

1c) yes

2a) yes

2b) no

3) $y = -\frac{3}{2}x + 9$

4) $y = -\frac{1}{3}x - 1$

Practice Questions

1a) perpendicular

1b) perpendicular

1c) neither

1d) parallel

2) $y = -4x + 2$

3) $y = -2.5x + 16.5$

Analytic Geometry Unit – Analytic Geometry Proofs

A proof is a true statement that must be justified (proven).

Analytic geometry proofs often involving proving a shape. These proofs require using

- The distance formula to find side lengths
- The rate of change to find parallel sides and right angles (perpendicular lines)

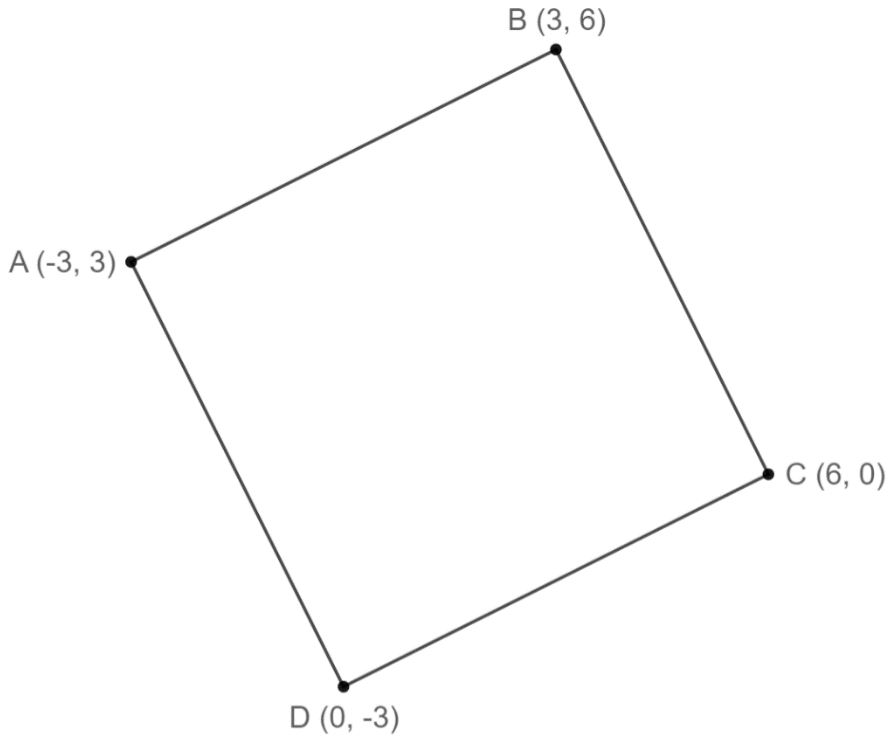
The following chart outlines some commonly used shapes and what you need to show in order to prove the shape.

SHAPE	SIDE LENGTH?	PARALLEL SIDES?	ANGLES?
Square	4 sides same length	n/a	At least one right angle
Rectangle	2 opposite sides same length 2 other opposite sides same length	n/a	At least one right angle
Parallelogram	2 opposite sides same length 2 other opposite sides same length	n/a	n/a
Rhombus	4 sides same length	n/a	No right angles
Trapezoid	n/a	Only 2 sides are parallel	n/a
Right Trapezoid	n/a	Only 2 sides are parallel	1 right angle
Isosceles Trapezoid	2 sides are the same length	Only 2 sides are parallel	n/a
Equilateral Triangle	3 sides are the same length	n/a	n/a
Isosceles Triangle	2 sides are the same length	n/a	n/a
Right Isosceles Triangle	2 sides are the same length	n/a	1 right angle
Scalene Triangle	Sides are all different lengths	n/a	n/a
Right Triangle	n/a	n/a	1 right angle

Note: n/a means this property does not apply in the definition of the shape.

To prove that a diagram shows a particular shape, you must show that the sides and angles satisfy the properties listed above.

Ex: Prove that the shape below is a square.



Try this question!

A rhombus has corners at $(-2, 6), (2, 7), (-3, 2), and\ (1, 3)$. Prove that this shape is a rhombus.